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ABSTRACT

This is one of a series of reports summarizing the results of the second mathematics assessment conducted by the National Assessment of Educational Progress (NAEP). Nine, 13- and 17-year-olds were assessed during the 1977-78 school year. The assessment measured achievement in various content areas at four levels of cognitive processes (knowledge, skill, understanding, and application). This report describes performance on knowledge and skill items. The report includes results for mathematical knowledge in the areas of numbers and numeration, geometry and measurement. The results for skills are divided into computational skills (computation with whole numbers, fractions, decimals, integers, percent, and conversion between fractional form) and other mathematical skills (measurement, reading graphs and tables, geometric manipulations, algebraic manipulations and estimation skills). The report also includes group results, age level comparisons and observations on the results. The observations, consisting of interpretation, consideration of implications, and recommendations, were made by a panel of persons active in the field. It was the judgment of this panel that while performance is generally satisfactory for knowledge and whole number computational skills, achievement in many other areas is below desired levels. Among the five recommendations is that the remedy for areas in which performance is not satisfactory should not be to expand rote drill and mechanistic teaching approaches but should strive to promote understanding. (PK)

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MATHEMATICAL KNOWLEDGE AND SKILLS

SELECTED RESULTS FROM THE SECOND ASSESSMENT OF MATHEMATICS

Report No. 09-MA-02

by the
National Assessment of Educational Progress

Education Commission of the States
Suite 700, 1860 Lincoln Street
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August 1979

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The cost figure cited above represents the total amount of money expended since late 1973 on assessments in art, career and occupational development, reading, writing, social studies/citizenship, science, basic life skills, mathematics and consumerism, resulting, to date, in numerous reports, papers, articles, presentations and assessment materials, many of which are used in state and local assessment programs. A complete list of all such materials is available upon request.

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FOREWORD

When the U.S. Office of Education was chartered in 1867, one charge to its commissioners was to determine the nation's progress in education. The National Assessment of Educational Progress (NAEP) was initiated a century later to address, in a systematic way, that charge.

Each year since 1969, National Assessment has gathered information about levels of education achievement across the country and reported its findings to the nation. NAEP surveys the education attainments of 9-year-olds, 13-year-olds and 17-year-olds in 10 learning areas: art, career and occupational development, citizenship, literature, mathematics, music, reading, science, social studies and writing. Different learning areas are assessed every year, and all areas are periodically reassessed in order to measure possible changes in education achievement. National Assessment has interviewed and tested more than 810,000 young Americans since 1969.

Learning-area assessments evolve from a consensus process. Each assessment is the product of several years of work by a great many educators, scholars and lay persons from all over the nation. Initially, these people design objectives for each subject area, proposing general goals they feel Americans should be achieving in the course of their education. After careful reviews, these objectives are given to exercise (item) writers, whose task it is to create measurement instruments appropriate to the objectives.

When the exercises have passed extensive reviews by subject-matter specialists, measurement experts and lay persons, they are administered to probability samples. The people who compose these samples are chosen in such a way that the results of their assessment can be generalized to an entire national population. That is, on the basis of the performance of about 2,500 9-year-olds on a given exercise, we can make generalizations about the probable performance of all 9-year-olds in the nation.

After assessment data have been collected, scored and analyzed, National Assessment publishes reports to disseminate the results as widely as possible. Not all exercises are released for publication. Because NAEP will readminister some of the same exercises in the future to determine whether the performance level of Americans has increased, remained stable or decreased, it is essential that they not be released in order to preserve the integrity of the study.

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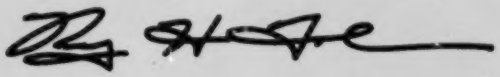
Many organizations and individuals have made substantial contributions to this report. Not the least of those to be gratefully acknowledged are the administrators, teachers and students who cooperated so generously during the collection of the data.

Special acknowledgment must go to Jane Armstrong of the National Assessment staff for coordination of development efforts and to Stuart Kahl and George Moreno for their assistance. Also to be acknowledged are the substantial contributions in guiding item development and report planning made by the members of the Mathematics Advisory Board: Dr. Sarah Herriot, Dr. Prince Jackson, Dr. Jeremy Kilpatrick, Dr. Donald Kreider, Dr. John LeBlanc and Dr. James Wilson.

Administration of the mathematics assessment was conducted by the Research Triangle Institute, Raleigh, North Carolina. Scoring and processing were carried out by the Measurement Research Center, Iowa City, Iowa, and by the National Assessment staff.

Mrs. Iris Carl, Dr. Thomas Carpenter, Dr. Robert Dilworth, Mr. Edgar Edwards Jr., and Dr. James Wilson contributed their time and knowledge interpreting the implications of the data. Their efforts helped greatly in putting the data into perspective.

The actual preparation of this report was a collaborative effort of the National Assessment staff. Special thanks must go to the following people: Ingrid Larsson and Charlotte Overbury for data processing support; Betty Homler for secretarial support; Ava Powell, Marci Reser, Eileen Wollam and Paula Pitchford for production. Statistical analyses for this report were supervised by Donald Phillips. The report was written by Stuart Kahl.



Roy H. Forbes
Director

INTRODUCTION

During the 1977–78 school year, the National Assessment of Educational Progress (NAEP) conducted its second mathematics assessment of the nation's 9-, 13- and 17-year-olds.[1] That assessment measured achievement in various content areas at four levels of cognitive processes — knowledge, skill, understanding and application. This report describes performance on knowledge and skill items.

The 1977–78 Mathematics Assessment

A two-dimensional matrix was used to guide the development of items for the 1977–78 assessment (*Mathematics Objectives, Second Assessment*, 1978, p. 17). Cognitive process levels formed one dimension of the matrix, and mathematical content areas, the other, as shown below.

Items were developed to assess students' performance with respect to each of the "cells" of this matrix. Over 700 items were used.

This report focuses upon mathematical knowledge and skill. Mathematical knowledge refers to the recall and recognition of mathematical ideas (such as facts and definitions) expressed in words, symbols or figures. It relies, for the most part, on memory and does not ordinarily require more complex mental processes. Examples of items assessing knowledge are recall of a basic computational fact (such as 2 + 2 = 4), recognition of the names of geometric shapes or recall of a mathemat-

[1] The first NAEP assessment of mathematics was completed in 1972–73; changes in achievement between 1973 and 1978 are described in *Changes in Mathematical Achievement, 1973–78* (1979).

PROCESS	CONTENT: Numbers and Numeration	Variables and Relationships	Geometry (Shape, Size and Position)	Measurement	Other Topics
Mathematical knowledge					
Mathematical skill					
Mathematical understanding					
Mathematical application					

ical relationship (for instance, the Pythagorean theorem).

Mathematical skill refers to routine manipulation of mathematical ideas. It relies on the ability to remember and use algorithms — that is, standard rules or procedures for finding an answer. Exercises assessing skill assume that the required algorithm has been learned and practiced, but they do not require that students decide which algorithms to use or apply an algorithm to a new situation. Skill items measured proficiency in using a particular algorithm rather than understanding how it works. Examples of tasks assessing skill are multiplying two numbers, making measurements, graphing a linear equation or bisecting an angle.

Developers of the mathematics assessment were careful to provide an appropriate balance between the content and process dimensions of the developmental matrix. To achieve this balance, the assessment was conceived of as an attempt to obtain data about the following questions:

Mathematical Knowledge

1. How well can students recall and recognize facts, definitions and symbols?

Mathematical Skill

1. How well can students perform computations, including computations with whole numbers, integers, fractions, decimals, percents, ratios and proportions?
2. How well can students make measurements?
3. How well can students read graphs and tables?
4. How well can students perform geometric manipulations like constructions and visualizations?
5. How well can students perform algebraic manipulations?
6. How well can students estimate the answers to computations and measurements?

Determining how "well" students perform on particular tasks is a subjective judgment. This report provides data about each of the questions listed and allows the reader to make his or her own judgment about students' performance.

Measuring Achievement

The mathematics assessment was administered to a national probability sample of 9-year-olds, 13-year-olds and 17-year-olds who were still attending school at the time of the assessment. The 9-year-olds were assessed in January and February of 1978; the 13-year-olds, in October through December of 1977; and the 17-year-olds, in March and April of 1978. About three-fourths of the 9-year-olds in the NAEP sample were enrolled in the 4th grade, and about one-fourth were in the 3rd grade. Around three-fourths of the 13-year-olds were in the 8th grade, and most of the remainder were in the 7th grade. Approximately 75% of the 17-year-olds were in the 11th grade; of the remainder, about 15% were in the 10th grade and about 10% were in the 12th.

National Assessment reports scores for groups of students, not individuals. It also provides estimated percentages of correct responses for single items. When a report indicates that "85% of the 17-year-olds gave a correct response," it means that an estimated 85% of the 17-year-olds would have given a correct response if all the 17-year-olds in schools across the country had been assessed. Therefore, it was not necessary for each student to respond to every item. Each respondent completed only one exercise booklet of about 45 minutes in length. Between 2,100 and 2,500 students responded to each booklet. In 1977—78, there were 7 booklets for 9-year-olds, 11 for 13-year-olds and 12 for 17-year-olds. Approximately 17,000 9-year-olds, 27,000 13-year-olds and

27,000 17-year-olds participated in the 1977—78 mathematics assessment.

About half of the exercises in the second mathematics assessment were multiple-choice and about half were open-ended, meaning that students had to supply an answer rather than choose a response from a list of alternatives.

Responses to open-ended items were tabulated in various categories. These categories were defined based on results of exercise pretests conducted prior to the actual assessment. Scoring categories included likely errors and thus are useful in identifying frequently made mistakes. To minimize guessing, students were encouraged to write "I don't know" on the answer line on open-ended items or to select the "I don't know" option on multiple-choice items if they felt they did not know the answer to a question. In an effort to prevent reading ability from affecting performance, exercises were recorded on tape and read aloud to respondents as well as printed in the item booklets.

Additional Information on the Mathematics Assessment

The following reports, available from National Assessment, describe other aspects of the NAEP mathematics assessment: *Mathematical Understanding* (forthcoming) reports 1977—78 results on items assessing understanding; *Mathematical Applications* (1979) describes students' performance in 1977—78 on application exercises; *Changes in Mathematical Achievement, 1973—78* (1979) discusses changes in ability between the first and second assessments of mathematics. *Mathematics Objectives, Second Assessment* (1978) describes the objectives and the processes by content matrix used to structure the 1977—78 assessment.

Technical documentation of the 1977—78 assessment is presented in a released exercise set showing items released for publication, with national and selected group results; a microfiche data appendix providing additional group results for released exercises; and a yearbook describing assessment procedures. Reports and data volumes describing results from the 1972—73 mathematics assessment are also available.

CHAPTER 1

MATHEMATICAL KNOWLEDGE

Numbers and Numeration

Mathematical competence at any level requires a basic knowledge of numbers and the decimal numeration system. Exercises dealing with naming numerals, ordering and place value are discussed in this section, along with results on exercises measuring knowledge of basic arithmetic facts and properties of numbers and operations.

Naming Numerals

Exercises in this category required the respondents to identify numerals for numbers written in words, and vice versa. It should be remembered that numbers written in words were also read to the respondents using paced-tape recordings for test administration. Nine-year-olds appear to have had little difficulty in naming whole numbers; 88% correctly wrote "five hundred twenty-two" using numerals. Given a two-digit and a three-digit whole number written in numerals, approximately 95% of the 9-year-olds selected the correct verbal expressions from a list of multiple-choice options.

Approximately 86% of the 9-year-olds could identify the numerical form of a common fraction from a list of options. In several free-response (open-ended) exercises, approximately 50% of the 9-year-olds correctly wrote common fractions in numerical form.

Results for exercises dealing with the naming of decimal fractions show large differences between age groups. In the exercise displayed in Exhibit 1, less than a fifth of the 9-year-olds identified "one and twenty-four hundredths," while the majority chose a whole number answer. These results are understandable, since many 9-year-olds have had little or no exposure to decimal fractions in school. Although the performance of 13-year-olds and 17-year-olds on this and a similar exercise was considerably higher than that of 9-year-olds, one-third of 13-year-olds and a quarter of 17-year-olds did *not* correctly identify the numerical form of the decimal number. On exercises requiring respondents to identify verbal forms of decimal fractions when given the numerals, performance was somewhat higher. Relative performance of the three groups was much as it was for the reverse task — approximately 24%, 80% and 92% were successful at ages 9, 13 and 17, respectively.

EXHIBIT 1. Percentage of Respondents Selecting Each Option of an Exercise on Naming Decimals, Ages 9, 13 and 17

Which of the following is one and twenty-four hundredths?

		Percent Responding		
		Age 9	Age 13	Age 17
○	1.024	11	14	15
●	1.24	19	65	76
○	2401	6	1	0
○	12400	57	18	7
○	I don't know.	6†	1†	1†

†Figures do not total 100% because of rounding and/or nonresponse.

Ordering

On three separate exercises, over 95% of the 9-year-olds identified the larger of 2 two-digit whole numbers. Many of the 9-year-olds (86%) and most of the 13-year-olds (96%) arranged 4 two-digit whole numbers in order from smallest to largest. Another exer-

cise required 9-year-old students to select the largest of 4 four-digit whole numbers that contained the same digits arranged in different orders. Three-quarters of the respondents correctly picked the largest.

Ordering fractions was somewhat more difficult. When asked to identify from a list of common fractions the one falling between two given common proper fractions, 81% of the 17-year-olds and 52% of the 13-year-olds made the correct selection.

Results on exercises dealing with the ordering of decimal fractions are consistent with the findings in the previous section on naming decimal fractions. The inexperience of students at age 9 was evident. Several exercises required the respondents to identify the decimal fraction falling between two given decimal fractions. About one-fourth of the 9-year-olds, two-fifths of the 13-year-olds and three-fourths of the 17-year-olds identified the correct decimal fractions. Exhibit 2 depicts the results for 13- and 17-year-olds on three exercises requiring respondents to select the greater or greatest of a set of decimal fractions.

In the second example, if one were to ignore the decimal point and assume the given numbers are whole numbers, one would arrive at the same correct answer. Thirteen-year-olds' and 17-year-olds' performance on this example was about the same. In the first and third examples, ignoring the decimal point would result in different answers; in these two examples, 17-year-olds' performance was substantially higher than that of 13-year-olds. Nevertheless, a fourth of the 17-year-olds failed to recognize that .2 was greater than the other decimal fractions given in the third example.

Place Value

Between 91 and 96% of the 9-year-olds correctly responded to free-response exercises involving only two-digit whole numerals, such as "What number is one more than 79?" The percentage of 9-year-olds who knew what number was 10 more than a given two- or three-digit number dropped to between 51 and 63%. Between 85 and 87% of the 13-year-olds responded correctly to the latter exercises.

Results on the two multiple-choice exercises reported in Exhibit 3 suggest that there is a large difference in 9- and 13-year-olds' abilities to identify the digit found in a particular "place" of a given whole number.

In two other multiple-choice exercises (not depicted in the exhibit) that asked what particular digits "stood for" in given numerals, the difference between 9-year-olds' and 13-year-olds' performance was somewhat smaller (80 and 87% for 9-year-olds versus 93 and 96% for 13-year-olds). Generally, when given the expanded notation for three-digit whole numerals in which place values are written in words (e.g., 2 hundreds + 5 tens + 5 ones), most 9-year-olds (83 to 90%) wrote the corresponding standard numeral. However, very few 9-year-olds (11 to 21%) performed this task when the expanded notation consisted entirely of mathematical symbols (e.g., 6 X 100 + 2 X 10 + 5 X 1) or when place values were "collapsed" (e.g., 16 tens + 5 ones, instead of 1 hundred + 6 tens + 5 ones).

Basic Arithmetic Facts

Knowledge of the basic arithmetic facts for addition, subtraction, multiplication and division is a prerequisite for any more advanced computation. In order to be sure students were responding on a recall basis, basic fact problems (e.g., 6 plus 3, 10 minus 6) were recorded on tape and played back to the respondents but did not appear in the test booklets. This allowed for a high degree of control over the response time for each basic fact, and also made it difficult, if not impossible, for students to go back to a basic fact exercise they did not complete in the few seconds allotted. Percentages of 9-year-olds responding correctly to various basic fact exercises are displayed in Exhibit 4. As

EXHIBIT 2. Percentages of Correct Responses on Selected Ordering Exercises, Ages 13 and 17

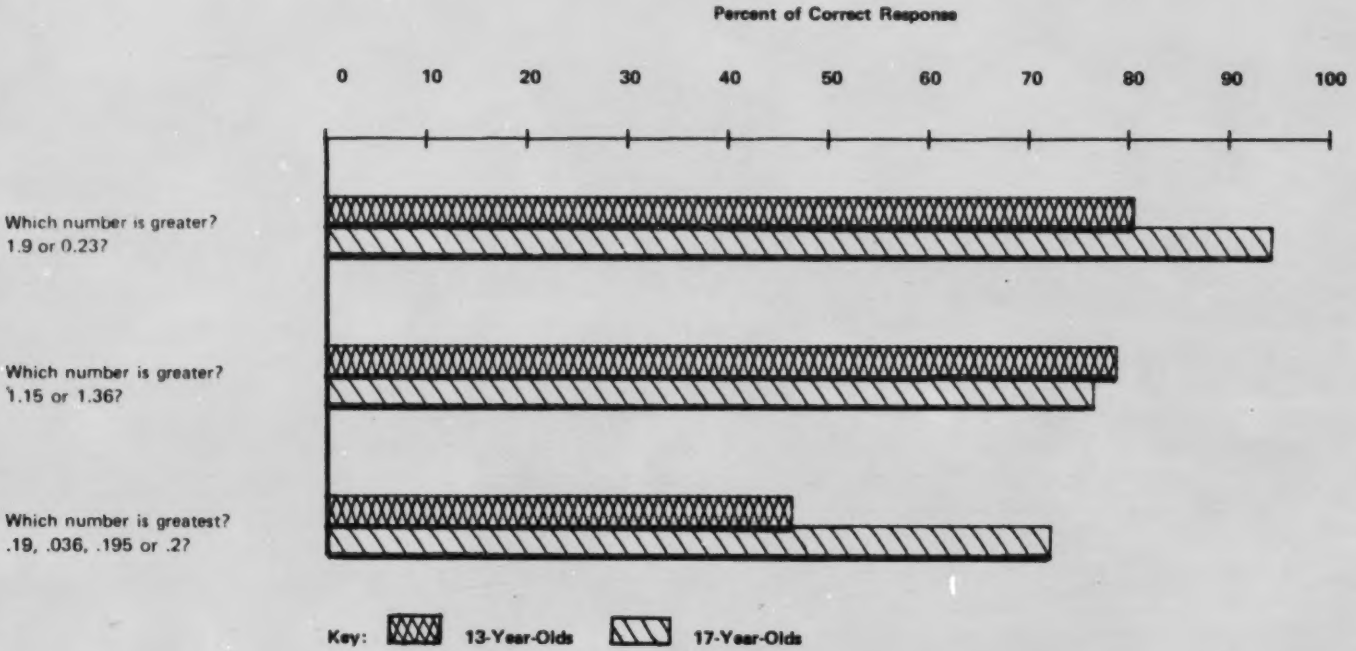
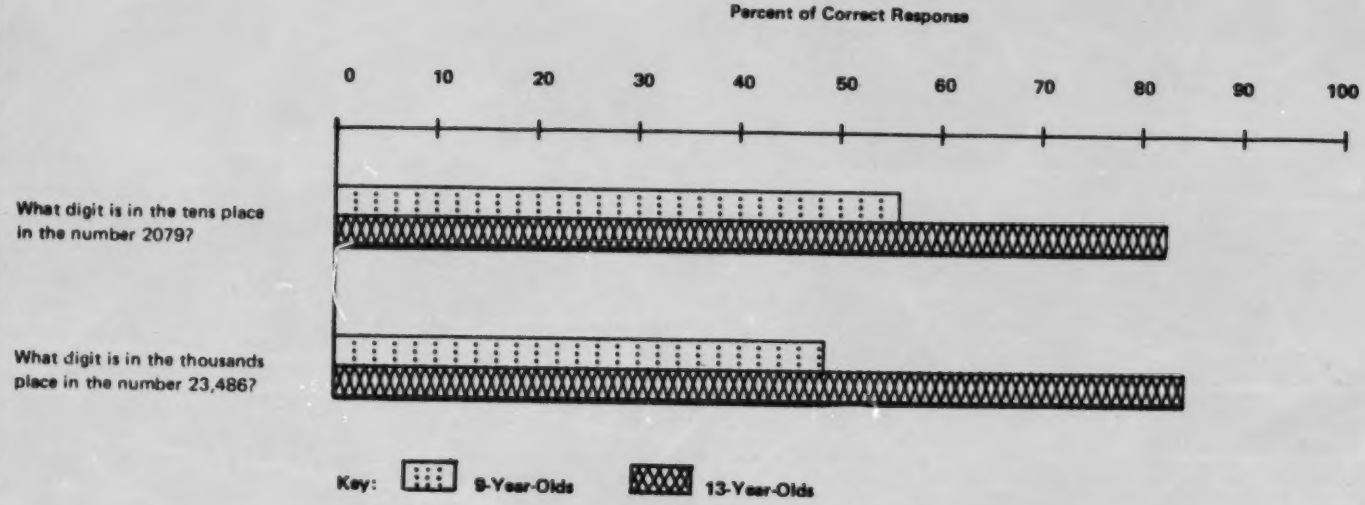


EXHIBIT 3. Percentages of Correct Responses on Selected Place-Value Exercises, Ages 9 and 13



expected, performance decreased as students proceeded from basic facts in addition to subtraction to multiplication. This order of operations corresponds with the order in which the operations are usually taught. Performance was lower on basic facts for each operation involving larger numbers (especially 9s) and on subtraction facts involving two-digit minuends (the numbers being subtracted *from*).

EXHIBIT 4. Percentages of 9-Year-Olds Responding Correctly to Selected Basic Fact Exercises

Fact	%	Fact	%	Fact	%
4 + 2	93	4 - 3	89	3 X 4	79
6 + 3	93	10 - 3	84	6 X 1	88
8 + 8	92	7 - 1	88	9 X 8	56
5 + 9	88	14 - 9	73	5 X 9	69
3 + 7	89	10 - 6	80	9 X 3	62
9 + 4	89	12 - 9	75	8 X 8	45

Several sets of basic fact exercises were administered to all three age groups. The average percent correct for each operation (addition, subtraction, multiplication and division) was computed; these are reported in Exhibit 5. As expected, these averages tend to increase with age, but decrease as one progresses from addition to division. The differences with respect to performance on the various operations are, of course, greatest for 9-year-olds, although at age 13 there is a large difference between performance on multiplication and division facts.

EXHIBIT 5. Average Percent Correct on Sets of Basic Fact Exercises Given to All Age Groups

	No. Items Average Based On	Average Percent Correct Age 9	Age 13	Age 17
Addition	6	89	95	97
Subtraction	6	79	93	95
Multiplication	6	60	93	93
Division	5	*	81	89

**Division facts not administered to 9-year-olds.*

Properties of Numbers and Operations

Some of the exercises in this category dealt with number theory. Given a list of four odd numbers and one even number, 93% of the 13-year-olds identified the even number. On another exercise, over 80% of the 13-year-olds knew what even numbers are. Fifty-eight percent of the 13-year-olds and 71% of the 17-year-olds recognized that a prime number is divisible *only* by itself and one, and 55% of the 13-year-olds and 62% of the 17-year-olds identified the prime number in a list of one- and two-digit numbers. On the latter exercise, approximately one-fifth of both age groups incorrectly labeled nine a prime number.

On two other number theory exercises, 72% of 13-year-olds identified a common factor of two given small numbers, while 24% of students at age 13 and 26% at age 17 gave the least common multiple of 3 one-digit numbers. The most common errors on these exercises suggest that students confuse factors and multiples.

Exhibit 6 shows the results for all three age groups on exercises involving properties of numbers and operations. Although respondents were given only a few seconds to answer each item, if they used the appropriate property in each case they could arrive at the correct answers fairly easily. The results on the third, fifth and sixth exercises indicate that few 9-year-olds were able to use the commutative and associative properties of addition and multiplication in these instances. Although the percentages of correct responses for the older age groups were substantially higher than those of the 9-year-olds, it appears that many students do not tend to use the properties even if they know them.

Geometry

During the last decade, mathematics instruction has placed increased emphasis on geometric concepts at all age levels — not just in formal geometry courses. Geometry exer-

EXHIBIT 6. Percentages of Correct Responses on Selected Exercises on Properties of Numbers and Operations, Ages 9, 13 and 17

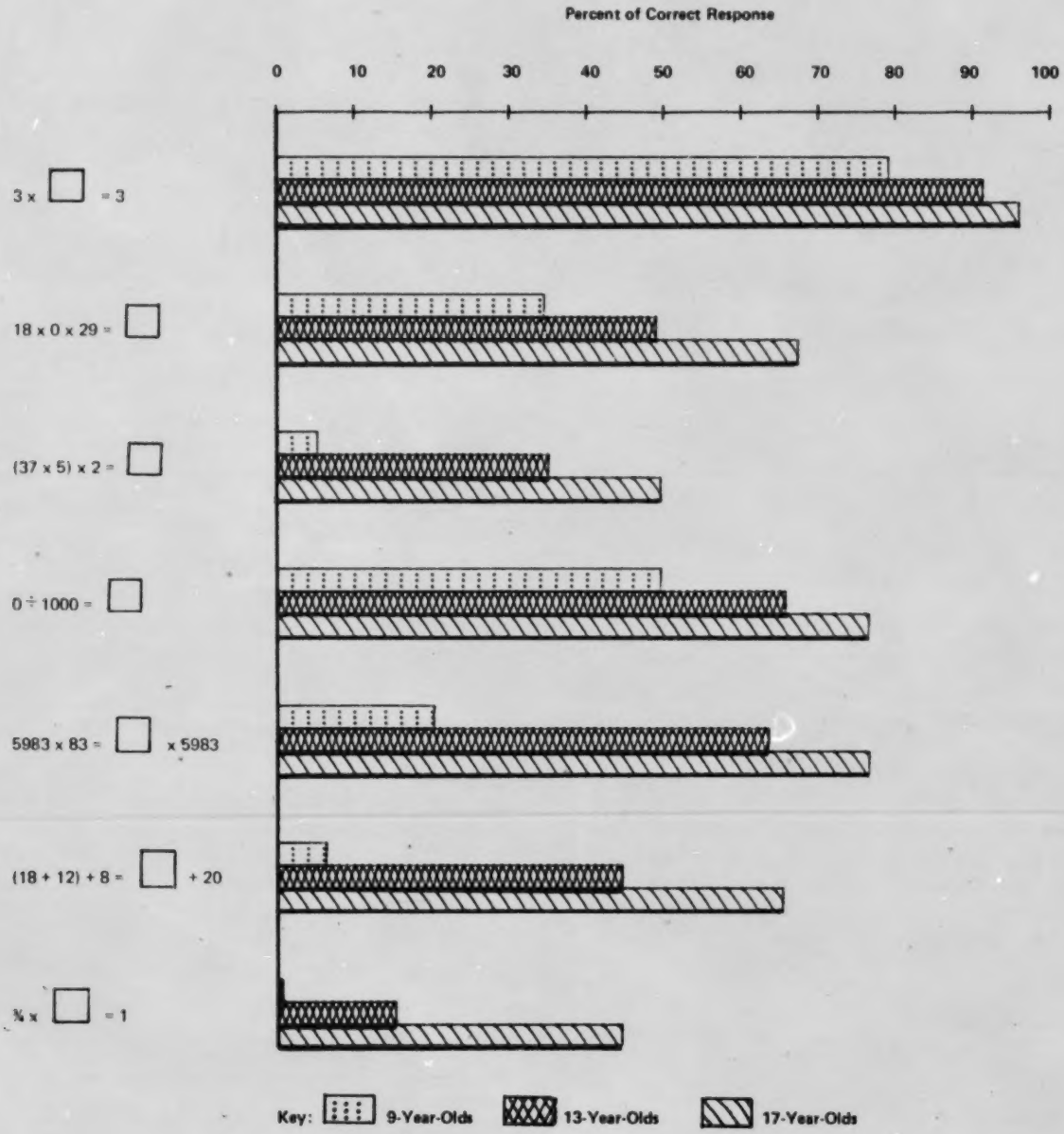
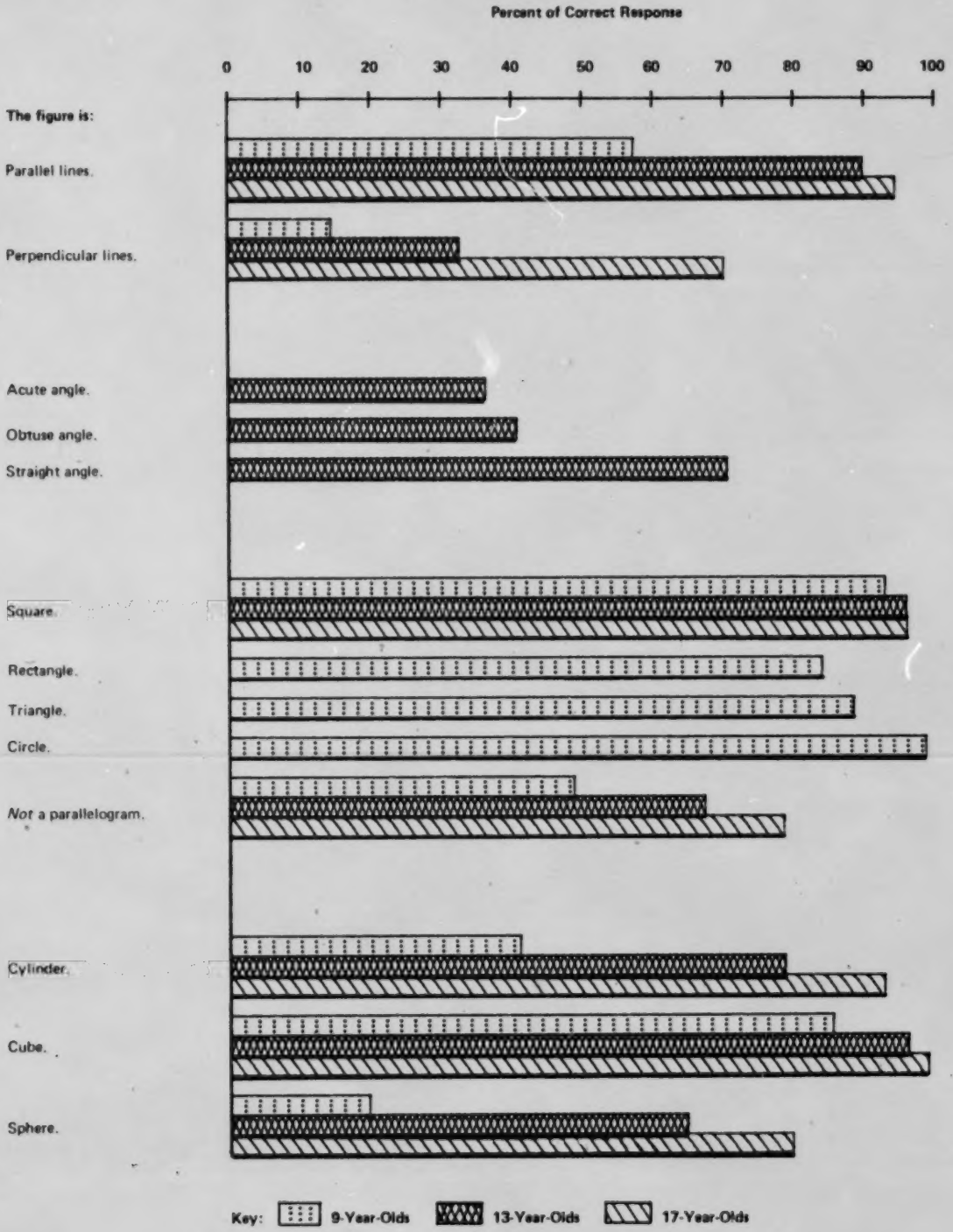


EXHIBIT 7. Percentages of Correct Responses on Exercises Dealing With Identification of Figures, Ages 9, 13 and 17



cises in the knowledge category measured students' ability to identify geometric figures and their knowledge of various characteristics of figures.

Identification of Figures

The exercises described in this section required respondents to match the name of a figure with a drawing of the figure. Exhibit 7 shows the percentages of correct responses for several exercises dealing with lines and angles, plane figures and solid figures. The knowledge of the two younger age groups varied widely depending on the figures. On an exercise not depicted in Exhibit 7, 21% of 9-year-olds, 50% of 13-year-olds and 73% of 17-year-olds knew that a table top is more like a geometric plane than is a water glass, an ice cream cone or a shoe box.

Characteristics of Figures and Their Parts

Although the recognition of figures does require some knowledge of their properties, the exercises discussed in this section measured specific knowledge of the characteristics of figures and their parts. Around 90% of 9-year-olds knew the number of angles in both a triangle and a square. Given a figure to refer to, 87% of 9-year-olds and 88% of 13-year-olds knew the number of sides in a rectangle; however, only 44% of 9-year-olds and 55% of 13-year-olds correctly gave the number of diagonals when shown a rectangle with diagonals drawn in. Given a figure illustrating corners, faces and edges of a cube, 63% of 9-year-olds, 84% of 13-year-olds and 90% of 17-year-olds determined the number of corners on the cube. Fewer students (42%, 75% and 84% at ages 9, 13 and 17, respectively) determined the number of faces. Percentages knowing the number of edges were about the same as those knowing the number of faces.

Exhibit 8 shows the responses of 13- and 17-year-olds on an item requiring the students to identify a line segment in a figure as being a radius.

EXHIBIT 8. Percentages of Respondents Recognizing a Radius, Ages 13 and 17

What is the dotted line in the circle called?

	Percent Responding	
	Age 13	Age 17
● radius	57	78
○ diameter	21	10
○ chord	5	4
○ tangent	3	2
○ I don't know.	14	5†

†Figures do not total 100% because of rounding and/or nonresponse.

In other exercises of this type, 32% of the 13-year-olds and 54% of the 17-year-olds correctly identified a chord, and similar percentages (35% of 13-year-olds and 59% of 17-year-olds) correctly identified a tangent. In two exercises given to 17-year-olds, 31% of them recognized a median of a triangle and 57% recognized a hypotenuse.

Some exercises dealt with more esoteric properties of plane figures. Twenty-four percent of 9-year-olds, 43% of 13-year-olds and 58% of 17-year-olds knew that if a triangle has two equal sides, then two angles must be equal. Six percent of the 13-year-olds and 31% of the 17-year-olds knew that the diagonals of a parallelogram always bisect each other, and 34% of the 17-year-olds knew that an altitude of a triangle is perpendicular to a side or its extension.

Measurement

Early in a child's school experience, after recognizing the need for standard units, a student must learn which measurement units are most appropriate in various situations. Furthermore, a knowledge of the relationships between units is essential in a number of everyday activities. Assessment exercises addressing these topics involved both English and metric units.

EXHIBIT 9. Percentages of Respondents Choosing Various Responses on Appropriate Metric Unit Exercise, Ages 9, 13 and 17

Which unit would you use to measure the length of your thumb?

	Percent Responding		
	Age 9	Age 13	Age 17
● centimeters	65	80	88
○ meters	18	11	6
○ kilometers	6	5	2
○ I don't know.	10†	4	3†

†Figures do not total 100% because of rounding and/or nonresponse.

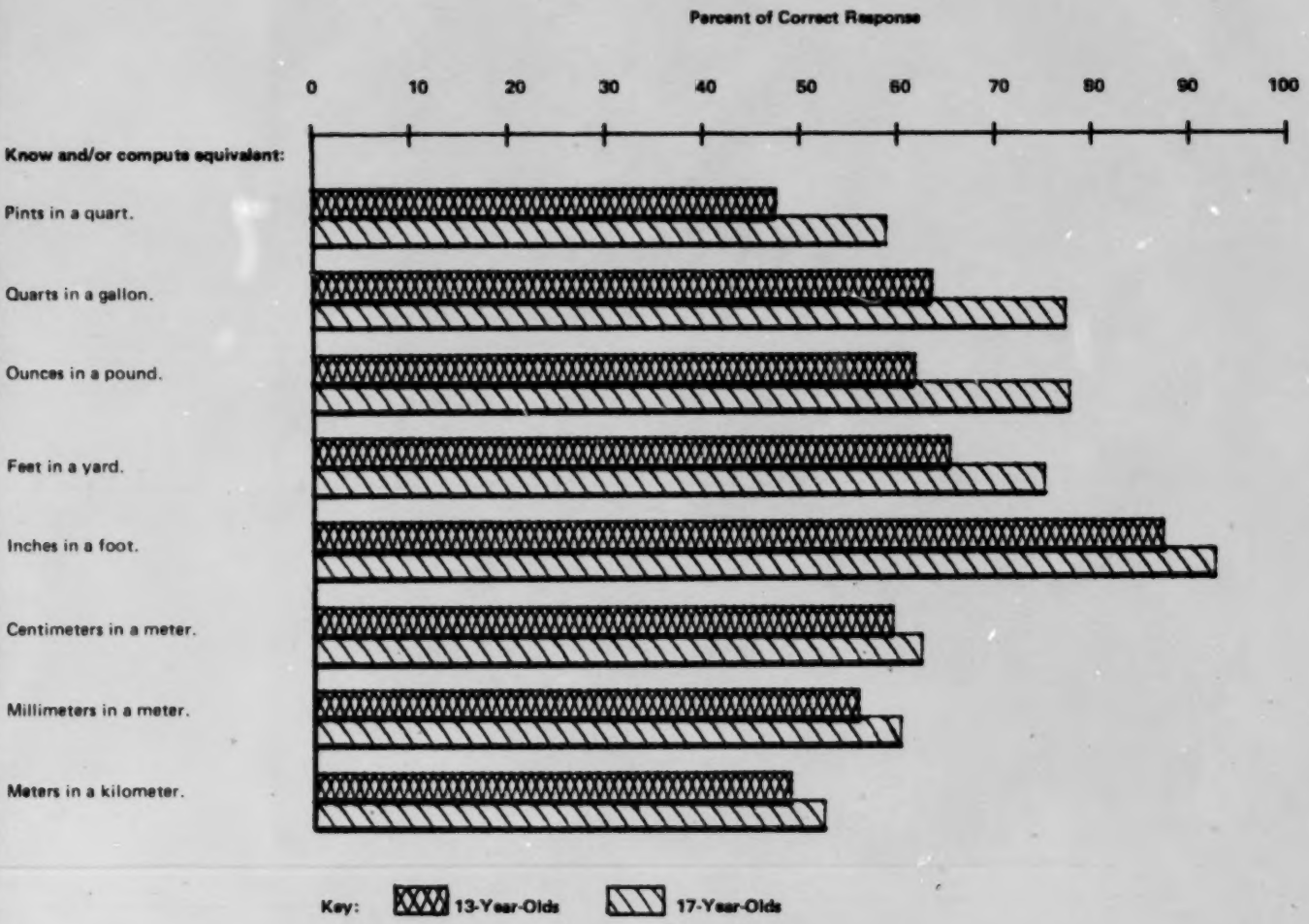
Appropriate Units

On one multiple-choice exercise, 88% of the students at age 9 knew that miles were the most appropriate English unit of linear measurement describing long distances. There were large differences between the percentage of 9-year-olds and the percentages of the other two age groups realizing that the gram, as opposed to other metric units, would be used for measuring weight (37%, 82% and 92% for ages 9, 13 and 17, respectively). Exhibit 9 shows the results for an exercise dealing with the appropriate metric unit of linear measurement for a given situation. The results on the exercises suggest that students do have some familiarity with the metric system.

Unit Equivalents

Exhibit 10 illustrates respondents' knowledge of unit equivalents. The exercises measuring this knowledge were all multiple-choice and asked how many of one unit are in another unit. Over a third of the 13-year-olds and a quarter of the 17-year-olds did not know that there are three feet in one yard.

EXHIBIT 10. Percentages of Correct Responses on Unit Equivalents Exercises, Ages 13 and 17



On an exercise not reported in Exhibit 10, 36% of the students at age 9, 63% at age 13 and 69% at age 17 correctly picked the largest of a set of metric units of length. Seventy-two percent of 17-year-olds knew which English unit was closest in length to a given metric unit.

Also included in the measurement category were exercises dealing with knowledge of coins and their value. Eighty-six percent of the students at age 9 recognized a picture of a dime and knew that it was worth 10¢. Exhibit 11 shows the results for an exercise that dealt with adding values of coins.

EXHIBIT 11. Percentages of 9-Year-Olds Choosing Various Responses on an Exercise Requiring the Addition of Coin Values

If you had 2 quarters, 1 dime and 3 nickels, how much would you have?

	Percent Responding Age 9
○ 50¢	6
○ 65¢	7
○ 70¢	4
● 75¢	68
○ 85¢	11
○ I don't know.	4

CHAPTER 2

COMPUTATIONAL SKILLS

Computation With Whole Numbers

There are some who feel that "basic skills" in mathematics translates to "computational skills." While many others would take issue with such a narrow definition of what is basic, few would exclude computational skills from a list of minimal competencies for all students. While people use computation in a variety of settings, a major use for adults is in job or consumer situations in which decimal fractions play an important part. The second mathematics assessment placed a great deal of emphasis on whole number computation. The algorithms for computing with whole numbers are the first taught to students, and they apply to computations with decimal fractions as well. The majority of exercises described in this section were open-ended.

The percentages of correct responses depicted in Exhibit 12 for selected addition exercises are high for the two older age groups. Over 80% of the 13-year-olds and at least 90% of the 17-year-olds answered these exercises correctly, even though some of the problems are fairly complex. The range of performance on these exercises was far greater for 9-year-olds than for 13- and 17-year-olds.

While 90% of the 9-year-olds correctly added 2 two-digit numbers when no regrouping (carrying) was required, only 50% correctly added a column of 4 two-digit numerals. It is interesting to note that on the exercise requiring respondents to find the "sum of 21 and 54," the percentages of correct responses for both 9- and 13-year-olds were approximately 15 percentage points less than they were for the exercise showing the same two numbers in alignment. Throughout

EXHIBIT 12. Percentages of Correct Responses on Whole Number Addition Exercises, Ages 9, 13 and 17

	Percent Correct		
	Age 9	Age 13	Age 17
21 +54	90	98	98
What is the sum of 21 and 54?	76	82	--
37 +18	76	95	97
43 71 75 +92	50	84	92
4285 3273 +5125	51	85	90

the assessment, different formats for presenting the same problems were used. Such related exercises were placed in different test booklets and thus were administered to different students. Generally, when students had to change the positions of numbers before computing or when technical vocabulary was involved (e.g., the word "sum" in this case), performance was considerably lower.

Results on the subtraction exercises shown in Exhibit 13 parallel those for the addition exercises; percentages of correct responses were fairly high for 13- and 17-year-olds. The greatest growth in computational ability oc-

curred between the ages of 9 and 13. Again, the percentage of correct responses dropped as complexity of the exercises increased, and the range of performance on the various exercises was greatest for 9-year-olds.

EXHIBIT 13. Percentages of Correct Responses on Whole Number Subtraction Exercises, Ages 9, 13 and 17

	Percent Correct		
	Age 9	Age 13	Age 17
2-digit number from 2-digit number, no regrouping (borrowing)	74	95	98
1-digit number from 2-digit number, one regrouping	58	92	96
2-digit number from 2-digit number, one regrouping	66	92	94
3-digit number from 3-digit number, two regroupings	50	85	92
3-digit number from 3-digit number, regrouping across two places (i.e., zero in minuend in tens place and regrouping required in ones place)	44	81	88
Subtract 237 from 504	28	73	84

Examining the incorrect responses of students on these exercises reveals two common errors worth noting. The most common error made by 9-year-olds was subtracting a smaller digit from a larger digit, regardless of whether or not the smaller digit was in the minuend. For example, a student making this error when subtracting 17 from 53 would get 44. The number of 9-year-olds making this error ranged from 8% to 32% on the exercises depicted in Exhibit 13. Very few 13 year-olds made this error; for them, regrouping (borrowing) errors were the most common. Regrouping errors were the second most common error for 9-year-olds.

Exhibit 14 shows the results on selected multiplication and division exercises. These results seem to reflect 9-year-olds' lack of experience with complex multiplication and division problems.

EXHIBIT 14. Percentages of Correct Responses on Whole Number Multiplication and Division Exercises, Ages 9, 13 and 17

	Percent Correct		
	Age 9	Age 13	Age 17
48 X4	38	--	--
1-digit number times 2-digit number, no regrouping (carrying)	65	95	97
1-digit number times 2-digit number with regrouping	34	84	92
2-digit number times 3-digit number, no regrouping	9	79	86
671 X402	3	66	76
2-digit number divided by 1-digit number, no remainder (basic division fact)	74	--	--
2-digit number divided by 1-digit number, no remainder (not a basic division fact)	19	--	--
$6\overline{)608}$	--	69	65
$28\overline{)3052}$	--	46	50

For the problem 402 X 671, 44% of the 9-year-olds tried to multiply within columns (decimal places) independently — for example, 2 X 1 is 2, 0 X 7 is 0 and 4 X 6 is 24, resulting in an answer of 2,402 or answers indicating a similar approach. Twenty-two percent of the 9-year-olds used such an approach on another exercise not shown in the exhibit.

About three-fourths of the 9-year-olds succeeded on a simple division task requiring only a knowledge of a basic division fact. However, on a slightly more difficult task, a two-digit number divided by a one-digit number with no remainder and not a basic fact, only about a fifth of the 9-year-olds were successful. The percentages of 13- and 17-year-olds responding correctly to the two long division exercises shown in Exhibit 14 were not very different.

A computational skill involving both addition and division is the computing of averages. On a multiple-choice exercise requiring students to compute the "average" of five whole numbers (4 one-digit numbers and 1 two-digit number), the percentages of correct responses were 6%, 51% and 71% for ages 9, 13 and 17, respectively. A different multiple-choice exercise asked 13-year-olds and 17-year-olds to compute the "mean" of the same numbers. Only 6% of 13-year-olds and 12% of 17-year-olds succeeded. Vocabulary appears to be an important factor related to performance on a particular task.

Computation With Fractions

Despite greater emphasis placed on the metric system and the increased importance of computation with decimal fractions associated with it, there are many who feel that the ability to compute with common fractions will not lose importance. Ordinary fractions are still likely to be used in many everyday activities. Furthermore, computations with fractions constitute less abstract cases of the algebraic manipulations taught in introductory algebra courses that are used to simplify algebraic expressions or to solve equations. Exercises concerning computation with fractions involve conversions between fractional forms as well as adding, subtracting, multiplying and dividing fractions.

Exhibit 15 shows some assessment exercises measuring students' ability to change fractions into equivalent fractional forms. The performance of 17-year-olds was about 15 to 20 percentage points higher than that for 13-year-olds on these exercises. Similarly, 17-year-olds did much better than 13-year-olds on exercises not shown in the exhibit requiring conversions of simple mixed numerals to improper fractions. Between 57 and 62% of the students at age 13 and between 80 and 83% of the students at age 17 answered these items correctly.

EXHIBIT 15. Percentages of Correct Responses on Exercises Dealing With Equivalent Fractions, Ages 13 and 17

Reduce the following fractions to lowest terms.

	Percent Correct	
	Age 13	Age 17
3/9 =	78	93
14/35 =	57	78
18/24 =	58	79

Change each of the following fractions to an equivalent fraction having the given denominator.

2/3 = ?/6	71	84
3/8 = ?/32	65	80

Write the following improper fractions as mixed numerals.

7/3 =	65	--
14/4 =	63	--

On related knowledge-level exercises for 13-year-olds, between 66 and 78% of the students correctly identified an improper fraction, a mixed numeral and a proper fraction. Knowledge of these terms was necessary to

solve several of the exercises previously described that required students to actually make the conversions between fractional forms.

Exhibit 16 shows the results on exercises dealing with addition and multiplication of fractions and mixed numerals. Very few of these exercises were administered to 9-year-olds because of their limited experience with such computations.

EXHIBIT 16. Percentages of Correct Responses on Addition and Multiplication of Fractions Exercises, Ages 9, 13 and 17

	Percent Correct		
	Age 9	Age 13	Age 17
Addition of fractions			
4/12 + 3/12 =	--	74	90
3/4 + 1/2 =	2	35	67
1/2 + 1/3 =	1	33	66
7/15 +4/9	--	39	54
2-2/5 + 5 =	--	44	66
2-3/5 +4-4/5	--	63	77
4-1/4 +3-2/5	--	46	60
Multiplication of fractions			
3/4 X 3/5 =	--	69	74
3-1/2 X 6-2/3 =	--	28	42
1/6 X 7-1/3 =	--	30	44

The most common error made in the addition of fractions is the adding of both the numerators *and* the denominators of the fractions. More 13-year-olds than 17-year-olds made this mistake. For example, in the exercise 3/4 + 1/2, 34% of the 13-year-olds and 15% of the 17-year-olds gave 4/6 or its equivalent as an answer. Twenty-three percent and 11% of the students at ages 13 and 17, respectively, gave 7-3/9 as an answer to 4-1/4 + 3-2/5.

On a knowledge-level item related to the problem 7/15 + 4/9, 24% of students at age 13 and 42% of students at age 17 could name the least common denominator of 7/15 and 4/9. Thus, a substantially larger number of students correctly performed the computation than were familiar with the terminology for a concept important to the process.

Results on exercises involving subtraction of fractions revealed errors similar to those on addition exercises. Incorrect responses indicate that students subtracted denominators of fractions. Eighty-one percent of 13-year-olds and 93% of 17-year-olds correctly subtracted one proper fraction from another when the two had a common denominator. Two subtraction exercises involved mixed numbers in which the fractions did not contain common denominators. On the one not requiring "borrowing," 35% of the 13-year-olds and 59% of the 17-year-olds succeeded. The percentages of correct responses on the other, which did require "borrowing," were 18% and 37% for students at ages 13 and 17, respectively.

A common error in the multiplication of mixed numerals was the separate treatment of whole numbers and fractions. For example, for the item 3-1/2 X 6-2/3, 27% of the 13-year-olds and 14% of the 17-year-olds multiplied 3 X 6 and then 1/2 X 2/3, arriving at 18-1/3. On the problem 3/4 X 3/5 and on three other exercises not shown in the exhibit dealing with the multiplication of proper fractions, some students divided the fractions (i.e., inverted the second fraction, then multiplied). A larger number of 17-year-olds than 13-year-olds made this error (6 to 9% at age 17 compared with 3 to 4% at age 13). On these same exercises, small percentages (2—4% of each age group) performed the initial

multiplications correctly, but gave as final answers responses indicating they reduced fractions improperly.

Although 9-year-olds have had limited experience with the multiplication of fractions, some exercises were administered to them requiring that they find a fraction of some whole number. While multiplication is the operation associated with these items, they could be solved without the use of the algorithm for multiplying fractions. Fifty-nine percent of the 9-year-olds knew 1/2 of 6, 29% knew 1/3 of 9, and 22% knew 1/4 of 8 and 2/3 of 9.

Computation With Decimals

Despite the similarities in algorithms for computations with whole numbers and decimals, the added burden of digits to the right of the decimal place generally resulted in lower performance on exercises dealing with the computation of decimals. In a previous section of this report, it was indicated that a large number of students were knowledgeable in the area of whole number place values, and that most students were able to name and/or write decimal numerals. Nevertheless, performance on computations with decimal fractions was considerably lower than performance on whole number computations. Analyses of the results of exercises in this section suggest that many students mechanically apply what they know about the processes involved in whole number computation to decimal fractions, sometimes ignoring the presence of decimal points completely.

EXHIBIT 17. Percentages of Correct Responses on Addition of Decimals Exercises, Ages 9, 13 and 17

	Percent Correct		
	Age 9	Age 13	Age 17
$.70 + $.40 + $.20	36	--	--
.70 + .40 + .20	11	58	79
.35 1.4	18	--	--
.35 + 1.4	1	52	77
7.54 1.52	32	--	--
7.54 + 1.52	15	72	88
3.57 + 1.2	--	56	80

Exhibit 17 shows the results on exercises dealing with the addition of decimal fractions. Some of these exercises involve the same numbers, but expressed in different formats.

Interestingly, 36% of the 9-year-olds correctly added the decimal fractions that were preceded by dollar signs, while only 11% correctly added the same decimal fractions when the dollar signs were omitted. Nine-year-olds have had some exposure to computations involving dollars and cents. However, money is treated in a whole number context in the early grades. When .35 and 1.4, already aligned for hand computation, were given to students to add, 18% of the 9-year-olds succeeded at the task, but only 1% succeeded when the same numerals were given in horizontal format. When faced with the horizontal format, over 50% of the 9-year-olds gave answers containing the digits 4 and 9 with various decimal placements. Thus, these students ignored fractional place values, added the 3 to the 1 and the 5 to the 4, then either added the decimal point somewhere or forgot about it. Although comparisons of the two formats for this exercise cannot be made for 13-year-olds and 17-year-olds since the vertical format was not given to those age groups, results on the exercise using the horizontal format show that older students made the same error as the 9-year-olds. Twenty-seven percent of 13-year-olds and 13% of 17-year-olds gave answers with the digits 4 and 9.

Results from the exercises in which 7.54 and 1.52 were to be added suggest that format is less of a problem when the two numbers involved have the same number of

digits to the right of the decimal point. However, 34%, 10% and 3% of the students at ages 9, 13 and 17, respectively, gave the right digits with the wrong decimal placement in response to 7.54 + 1.52 shown horizontally. Twenty-nine percent of the 9-year-olds gave such responses even when the numerals were aligned vertically.

Exercises given to 13- and 17-year-olds involving subtraction of decimals resulted in similar patterns of errors. Again, performance was lower when the numbers differed in the number of places to the right of the decimal point, especially when the horizontal format was used.

On four decimal-multiplication exercises involving one- or two-digit factors, the percentages of correct responses for 13-year-olds ranged from 46% to 73% and for 17-year-olds, from 68% to 87%. Since aligning decimal points is not a concern in multiplication, most incorrect responses contained the correct digits, but had the wrong decimal placement.

Similar errors were made by 13- and 17-year-olds on three exercises involving division of decimals. That is, the most common error was improper placement of the decimal point in the answer. On an exercise involving a one-digit whole number divisor and a two-digit dividend with one digit to the right of the decimal, about three-fourths of the 13-year-olds and 90% of the 17-year-olds succeeded. On an exercise involving the same digits as the previous example, except that the divisor was a decimal fraction, the percentages of correct responses for 13- and 17-year-olds were 34% and 52%, respectively. These results and similar results for another exercise indicate many students do not know how to deal with decimals in a divisor.

Computation With Integers

The rules for computing with integers are directly applicable to more abstract algebraic manipulations. Computation with integers is introduced prior to formal instruction in algebra. Exhibit 18 shows the possible combinations of positive and negative integers for each of the four operations. The actual integers used in these exercises were small so that success on these exercises would depend more on students' knowledge of the rules for integer computation than on their skill with the operations.

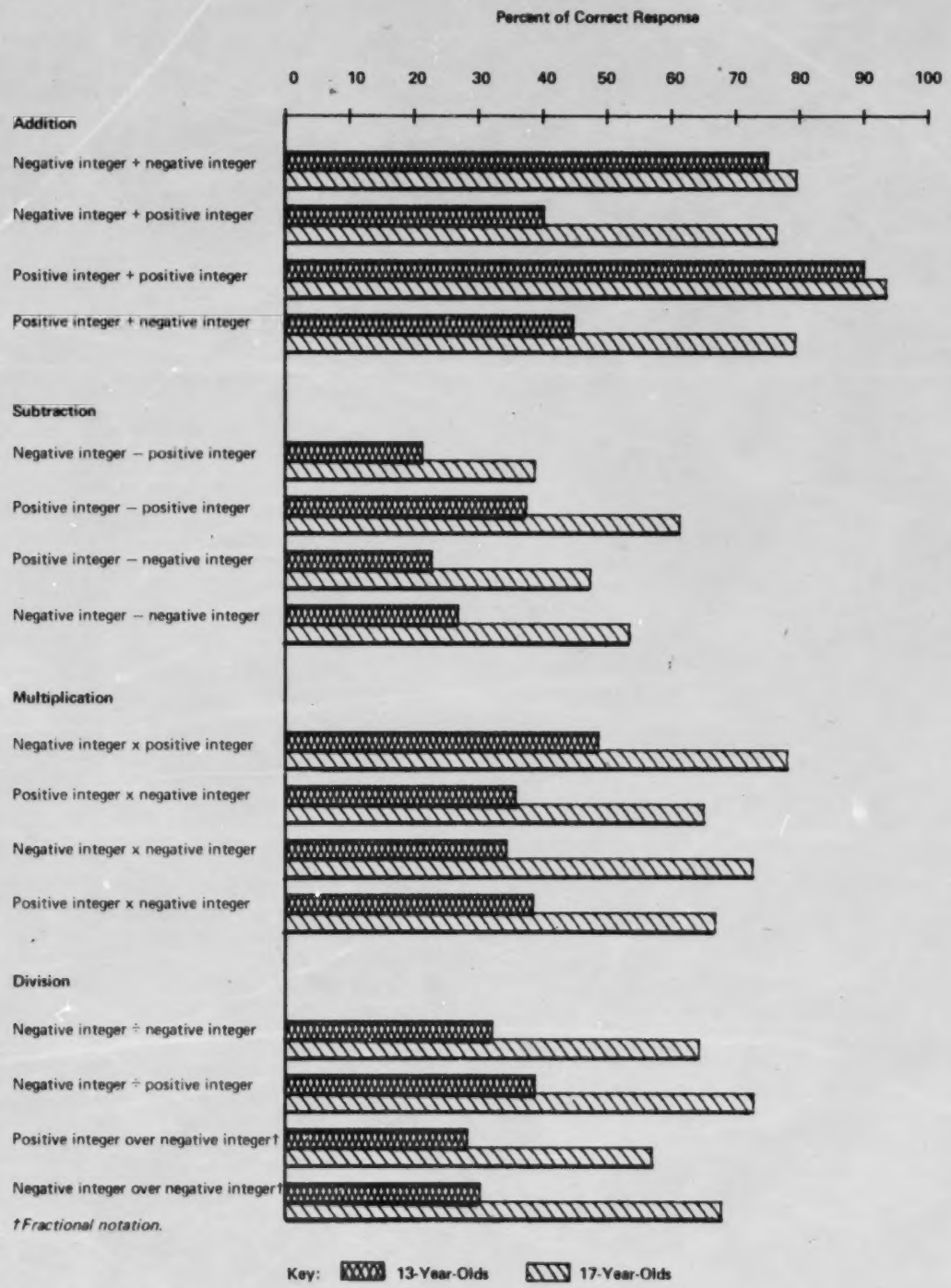
In most cases, the percentages of correct responses for 17-year-olds were 20 to 30 percentage points greater than those for 13-year-olds. Two notable exceptions involved the addition of two negative integers and two positive integers. Considering the "rules" for computing with integers, one would expect a student with little or no experience in integer computation to be more likely to guess the correct answer in those instances. Although computation with integers is generally introduced prior to age 13 and high school students probably do not spend a great deal of time with it, the experience of older students with algebraic manipulations might account for their superior performance on these exercises.

Examples of the integer computation exercises are given in Exhibit 19. On another exercise not shown in the exhibit, 36% of students at age 13 and 66% of students at age 17 knew that the product of two integers is negative when one is negative and one is positive. These percentages are almost identical to the percentages of correct responses for the second multiplication exercise in the exhibit.

Computations Involving Percent

Percentage has traditionally been a difficult concept for students to deal with at any cognitive level. Results on the percentage computation exercises in the mathematics assessment suggest that this is still the case. Percentage exercises involve three components — a whole, a part and a percentage rate. Given two of these components, students were asked to determine the third. Exhibit 20 depicts the results of five such exercises. The numbers used in these exercises

EXHIBIT 18. Percentages of Correct Responses on Computations With Positive and Negative Integers, Ages 13 and 17



†*Fractional notation.*

EXHIBIT 19. Percentages of Responses on Two Integer Computation Exercises, Ages 13 and 17

	Percent Responding	
	Age 13	Age 17
$^{-}4 - {}^{+}7 =$		
○ $^{+}3$	40	43
○ $^{-}3$	16	9
○ $^{+}11$	11	6
● $^{-}11$	21	39
○ I don't know.	12	2†
$^{+}7 \times {}^{-}7 =$		
○ $^{+}49$	23	14
● $^{-}49$	36	65
○ 0	20	16
○ $^{+}14$	7	2
○ $^{-}14$	4	1
○ I don't know.	9†	2

†Figures do not total 100% because of rounding and/or nonresponse.

EXHIBIT 20. Percentages of Correct Responses on Selected Exercises on Percent, Ages 13 and 17

	Percent Correct	
	Age 13	Age 17
30 is what percent of 60?	35	58
What is 4% of 75?	8	27
12 is 15% of what number?	4	12
What is 125% of 40?	12	31
6 is what percent of 120?	6	16

are fairly simple ones with which to work.

Normally, exercises at the skill level can be solved by the "mechanical" use of algorithms and require no understanding of why the algorithm works or of a particular problem situation. Although the exercises shown in Exhibit 20 are normally assigned to the skill level, it may well be that such computations are exceptions to the definition of skill exercises. To solve these exercises mechanically, a student must determine which of the three elements is missing, then choose an algorithmic strategy depending on which element is to be found. Because of the simplicity of the numbers, students with a good understanding of percentage would be able to answer such questions without using tedious, mechanical approaches. Results on another exercise not shown in the exhibit show that 49% of 13-year-olds and 58% of 17-year-olds knew that more than 100% of a number is greater than that number and that similar percentages of those same age groups (47% and 68%, respectively) knew that less than 100% of a number is less than that number.

Conversions Between Fractional Forms

Various everyday activities, especially in consumer settings, either require people to make conversions between fractional forms or would be simplified by the use of such conversions. One set of unreleased exercises asked students to write three proper fractions and one mixed numeral with denominators of 10 or 100 as decimals. The percentages of 13-year-olds answering these problems correctly ranged from 55 to 64%, and the range of percentages for 17-year-olds was from 69 to 82%. Forty-one percent of 13-year-olds and 59% of 17-year-olds correctly converted an improper fraction with 100 as the denominator to its decimal form. Less than 1% of 9-year-olds responded correctly to any of these exercises.

Three multiple-choice exercises required students to select the decimal equivalent of common proper fractions. In the simplest case, in which a large number of students would most likely have memorized the decimal equivalent, 38% of students at age 13 and 75% at age 17 succeeded. About a fourth of the 13-year-olds and a little over half of the 17-year-olds were successful in the other two cases that were slightly more complex and would require most students to "work them out."

Exhibit 21 depicts the results on six other exercises involving conversion between fractional forms. For most of these, approxi-

mately 20% more 17-year-olds than 13-year-olds responded correctly. Unlike the percentage exercises described in the previous section, conversions between fractional forms involving percentages can be performed using simple algorithms previously memorized (e.g., simply moving the decimal point). Consequently, these computations involving percent result in higher percentages of correct responses than those in the previous section.

EXHIBIT 21. Percentages of Correct Responses on Conversions Between Fractional Forms Exercises, Ages 13 and 17

	Percent Correct	
	Age 13	Age 17
Change decimal fraction with one nonzero digit to the right of the decimal point to common fraction	53	72
Change decimal fraction with two nonzero digits to the right of the decimal point to common fraction	49	68
Change decimal fraction with nonzero digits in hundredths and thousandths place to common fraction	43	60
Change 25% to a common fraction	57	81
Change 125% to a decimal fraction	27	44
Change .15 to a percent	68	77

CHAPTER 3

OTHER MATHEMATICAL SKILLS

Introduction

In Chapter 2, the achievements of students with respect to computational skills were described. There are many important mathematical skills in addition to computation. In this chapter, performance of students with respect to measurement skills, reading graphs and tables, geometric manipulations, algebraic manipulations and estimation skills are discussed.

Measurement Skills

Making Measurements

The exercises in this category deal with reading measurement instruments and making linear measurements. Three multiple-choice exercises administered only to 9-year-olds required the students to read the time on a clock showing an even hour, 15 minutes past that same hour and 40 minutes past that same hour. The percentages of correct responses for those tasks were 93%, 86% and 68%, respectively. Exhibit 22 shows the results for 9- and 13-year-olds on a slightly more difficult task dealing with reading time. These four exercises taken collectively show that, for 9-year-olds, performance decreased as the tasks required finer and finer distinctions. Notice that an incorrect option chosen by 15% of the 9-year-olds on the exercise shown in Exhibit 22 was an even multiple of 10 minutes before the hour.

EXHIBIT 22. Percentages of Responses on Reading Time Exercise, Ages 9 and 13

What time is shown on the clock?

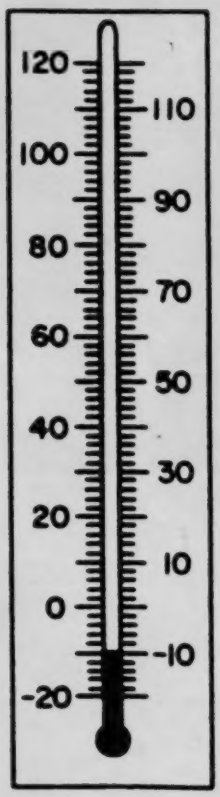
	Percent Responding	
	Age 9	Age 13
○ 12:43	10	5
● 12:48	59	85
○ 12:50	15	5
○ 12:52	8	3
○ I don't know.	8	1†

†Figures do not total 100% because of rounding and/or nonresponse.

Eighty-one percent of the 9-year-olds correctly read a bathroom scale that used one-pound graduations. Approximately the same percentage of 9-year-olds correctly read the thermometer shown in Exhibit 23. Close to 90% at age 13 and over 90% at age 17 read it correctly. The incorrect option chosen most frequently on this exercise was 10 degrees. It

may be that when reading temperatures the younger students are more familiar with the expression "10 degrees below zero" than with the negative integer notation.

EXHIBIT 23. Percentages of Responses on Reading a Thermometer Exercise, Ages 9, 13 and 17



What temperature is shown on this thermometer?

		Percent Responding		
		Age 9	Age 13	Age 17
●	-10°	80	89	94
○	-5°	0	0	0
○	5°	0	0	0
○	10°	18	10	5
○	I don't know.	1†	0†	0†

†Figures do not total 100% because of rounding and/or nonresponse.

Another exercise required the students to read a temperature that was above zero on the same type of thermometer. In that exercise, however, the thermometer did not show a temperature that was an even multiple of 10 degrees. Twelve percent of the 9-year-olds, 46% of the 13-year-olds and 62% of the 17-year-olds selected the correct temperature. The two-degree graduations on that thermometer apparently posed some difficulty for students, as large numbers of students (80% at age 9, 53% at age 13 and 37% at age 17) chose responses one would obtain by counting graduations by ones instead of twos.

Several assessment exercises dealt with linear measurement. One exercise required each student, supplied with a paper clip, to determine how many paper clips long a particular line segment was. On this exercise, involving measurement using nonstandard units, 65% of 9-year-olds, 85% of 13-year-olds and 89% of 17-year-olds succeeded. The exercise shown in Exhibit 24 is illustrative of a set of exercises used to test students' abilities to "read" a ruler.

On a similar exercise requiring students to determine the length of an object to the nearest inch, just over three-fourths of 9- and 13-year-olds supplied the correct answer. On still another exercise of this type, 68% of 9-year-olds correctly determined the length of an object to the nearest centimeter. Thus, as one might expect, performance was higher for gross measurements than on measurement exercises requiring finer distinctions.

On another exercise, the end of the object to be measured was not placed at the end of the ruler in the drawing. On this exercise, which was multiple-choice, only 19% of 9-year-olds and 59% of 13-year-olds selected the correct answer. About three-fourths and two-fifths of the students at ages 9 and 13, respectively, incorrectly selected the option representing the point on the ruler where the right-end point of the object was located.

Exhibit 25 depicts the results on exercises

EXHIBIT 24. Percentages of Correct Responses on Linear Measurement Exercise, Ages 13 and 17

What is the length of this pencil to the nearest quarter inch?

Answer (4¾) inches

Percent Correct	
Age 13	Age 17
53	81

requiring students to measure line segments using actual rulers. Performance was substantially lower on the two exercises requiring the use of metric rulers than it was on the exercise requiring the use of an English ruler. Of course, the second of the metric exercises presented a more complicated task in that students had to make three measurements, then add the three measures together. For the exercises on which students at age 9 outperformed those at age 13, a large number of 13-year-olds actually gave a more precise answer, but failed to round to the nearest centimeter. Another exercise asked students, equipped with a metric ruler, to draw a line segment of a particular length (a whole number of units). Sixty-seven percent and 74% of the students at ages 9 and 13, respectively, succeeded at this task.

EXHIBIT 25. Percentages of Correct Responses on Measuring Line Segments Using Rulers Exercises, Ages 9, 13 and 17

	Percent Correct		
	Age 9	Age 13	Age 17
Find length of line segment to nearest inch	80	91	94
Find length of line segment to nearest centimeter	62	54	--
Find distance around triangle in centimeters	38	65	71

Conversions Between Measurement Units

Exercises in this category required students to convert measures expressed using one unit to measures expressed using another unit. The results on several of these exercises are given in Exhibit 26. These results can best be summarized by saying that large numbers of students in all three age groups could not deal with these exercises. (The largest percentage of correct responses for 17-year-olds was 58.) It should be remembered that in order to succeed on these exercises students must have a knowledge of the applicable unit equivalents, a knowledge shown in Chapter 1 of this report to be lacking. Thus, it can be concluded that a large number of the students who failed to respond correctly to these exercises probably did so because they did not have this prerequisite knowledge. Many of

EXHIBIT 26. Percentages of Correct Responses on Exercises Involving Conversions Between Units, Ages 9, 13 and 17

	Percent Correct		
	Age 9	Age 13	Age 17
How many feet in 60 inches?	11	60	--
inches in 6 feet, 5 inches?	--	47	--
yards in x feet?	--	43	58
square yards in x square feet?		--	8
centimeters in 5 meters, 34 centimeters (multiple choice)?	28	40	52
meters in x centimeters (multiple choice)?	--	44	52
ounces in x pounds?	--	33	56
pints in x gallons?	--	41	49

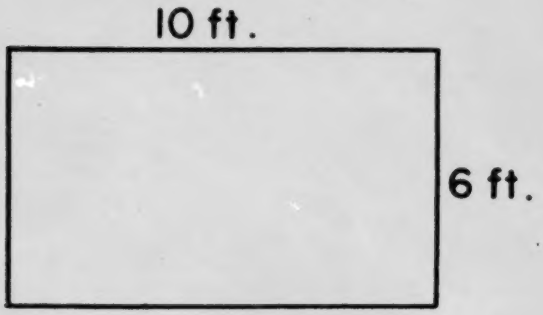
NOTE: For unreleased exercises described in this exhibit, the place holder X was used instead of the actual numbers that appeared in the exercises.

the items in Exhibit 26 were presented in a word-problem format and thus required some problem-solving or application abilities as well as skill in unit conversions.

Perimeter, Area, Volume

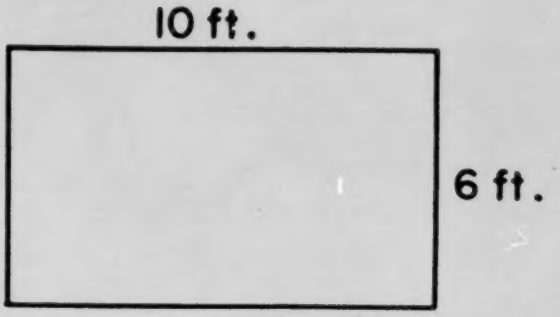
Results on the two perimeter exercises shown in Exhibit 27 illustrate again the effect of vocabulary on the performance of a mathematical task. Less than 10% of the 9-year-olds and less than one-half of the 13-year-olds found the "perimeter" of a rectangle, but 40% of the 9-year-olds and 69% of the 13-year-olds found the "distance all the way around" the same rectangle. Thus, vocabulary is an important consideration when one is evaluating the achievements of students. It appears that if students were faced with the task of determining how much molding would be required to lay around a room, many more students would be able to do so than might be indicated by the exercise results using the technical term *perimeter*.

EXHIBIT 27. Percentages of Responses on Two Perimeter Exercises, Ages 9 and 13



What is the PERIMETER of this rectangle?

	Percent Responding	
	Age 9	Age 13
○ 16 feet	66	25
○ 30 feet	2	1
● 32 feet	8	49
○ 36 feet	2	4
○ 60 feet	3	17
○ I don't know.	16†	4



What is the DISTANCE ALL THE WAY AROUND this rectangle?

	Percent Responding	
	Age 9	Age 13
○ 16 feet	39	12
○ 30 feet	4	1
● 32 feet	40	69
○ 36 feet	4	4
○ 60 feet	4	13
○ I don't know.	8†	1

†Figures do not total 100% because of rounding and/or nonresponse.

Exhibit 28 summarizes students' performance in computing areas of figures. Performance was substantially higher for figures in which unit squares were drawn than for exercises involving figures that were no more complex but in which unit squares were not shown. On these exercises, students with little or no facility with the mathematical computation of area could still count unit squares provided they had some knowledge of the concept of area. When unit squares are not drawn, students would most likely rely on the mechanical application of a previously memorized formula. Notice on the second and third exercises shown in the exhibit that the performance of 13-year-olds was 20 percentage points lower for the figure without unit squares.

Thirty-three percent of the 13-year-olds successfully computed the volume of a rectangular solid, the dimensions of which were small, one-digit whole numbers. About half

EXHIBIT 28. Percentages of Correct Responses on Computing Area Exercises, Ages 9, 13 and 17

	Percent Correct		
	Age 9	Age 13	Age 17
Area of figure consisting of square units and halves of square units (multiple choice)	--	76	88
Area of a rectangle given whole number dimensions with unit squares drawn in (multiple choice)	28	71	--
Area of same rectangle as in previous exercise without unit squares drawn in (multiple choice)	4	51	74
Area of square given length of a side	--	12	42
Area of right triangle given length of all three sides	--	4	18
Area of parallelogram given lengths of sides and altitude	--	--	19

that number of 13-year-olds (16%) found the volume of a rectangular solid with larger two-digit dimensions. Thirty-nine percent of the 17-year-olds succeeded on the latter task.

Reading Graphs and Tables

One need only glance at a newspaper or a weekly news magazine to realize that a tremendous amount of information is presented to the public in graphs and tables. Certainly an informed citizen and consumer must be able to read and interpret information presented in such forms.

National Assessment exercises in this category involve circle, bar and line graphs as well as tables in various forms. Most of these exercises include multiple parts, and in many cases, the parts of an exercise address different cognitive process levels. Sometimes a student may be required merely to make a direct reading from a graph or table. Sometimes an interpretation, either simple or complex, may be required. For example, an item may ask, "On what day did the most rain fall?" or "Which car was moving the fastest 10 seconds after it started?" Problem-solving exercises might require students to use multiple readings from a graph or table and/or other information presented in the exercise in order to solve the problem. Direct reading of graphs and tables is considered a skill, and for that reason, exercises that mostly involve direct reading are discussed in this report. However, all of the parts of an exercise are discussed, even if some of them involve higher cognitive processes. This section emphasizes exercises that have been released, since it is difficult to paraphrase unreleased exercises of this type in specific enough terms for the results to be understandable.

In the problem shown in Exhibit 29, 9-year-olds were supposed to select the graph that depicts the data presented in the item stem. More students selected the first option, in which the bars increase in length as the ages of the boys increase, rather than the correct option (36% compared with 29%). The first option would be an obvious choice for students who did not understand what the vertical axis represents.

Exhibit 30 shows a circle graph exercise that was administered to 13- and 17-year-olds. Part A might be said to require a very simple interpretation, while part B clearly requires a direct reading. For both age groups, performance on those two parts was far superior to that on the third part, which would be considered application-level.

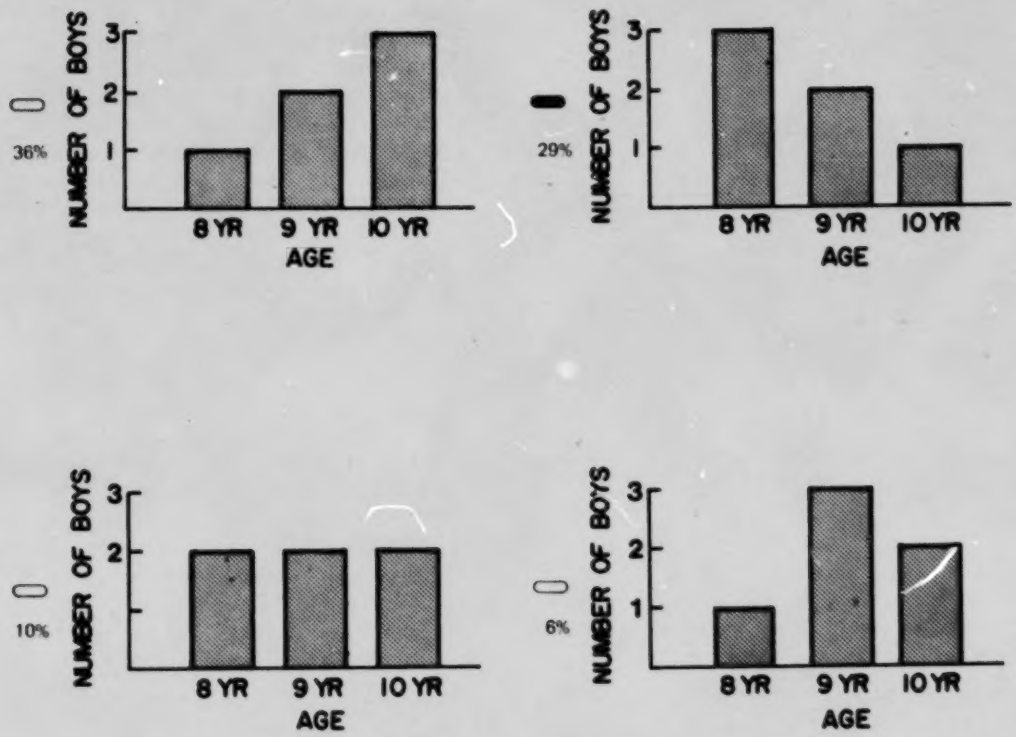
The line graph shown in Exhibit 31 is typical of one that might appear in the popular press. Eighty-seven percent of the 17-year-olds selected the correct answer to part A, which required a direct reading of the graph. Part B required two readings and a simple computation, and two-thirds of the 17-year-olds succeeded at this task. A little

EXHIBIT 29. Percentages of Responses on Bar Graph Exercise, Age 9

Here are the ages of six boys:

10, 9, 9, 8, 8, 8

Which bar graph presents this information?



36%

29%

10%

6%

16%†

I don't know.

†Figures do not total 100% because of rounding and/or nonresponse.

more than a third succeeded in the application situation presented in part C.

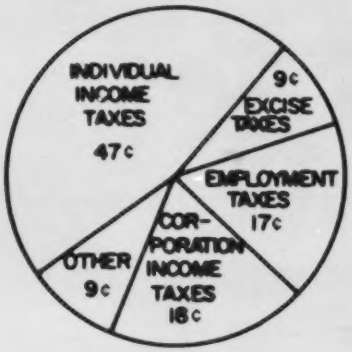
Exhibit 32 depicts the results on two unreleased exercises asking the same questions and using the same data presented in different formats — a circle graph in one case and a pictograph in the other.

The direct-reading part of the circle graph exercise merely required students to find the appropriate labeled section of the circle and identify the number that was written in that section. In the pictograph, frequencies of individuals were represented by symbols, and each symbol stood for more than one individual. This would account for the lower

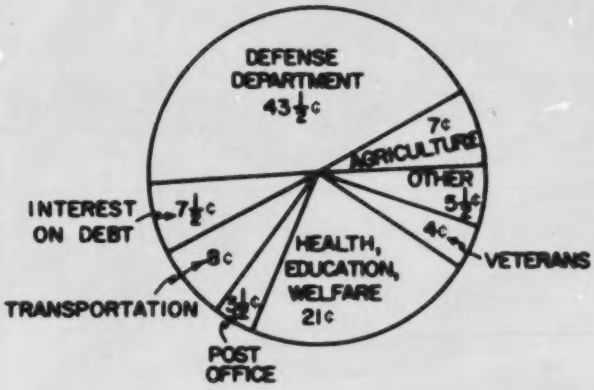
EXHIBIT 30. Percentages of Responses on Circle Graph Exercise, Ages 13 and 17

THE FEDERAL GOVERNMENT DOLLAR

WHERE IT COMES FROM



WHERE IT GOES



		Percent Responding Age 13	Age 17
A.	Where does the largest part of the Federal Government Dollar come from?		
○	Excise taxes	2	0
○	Employment taxes	3	3
○	Corporation income taxes	5	1
●	Individual income taxes	81	93
○	Defense Department	4	2
○	I don't know.	5	1
B.	What part of the Federal Government Dollar is spent on Agriculture?		
●	7¢	89	95
○	21¢	2	0
○	28½¢	2	0
○	43½¢	2	0
○	47¢	1	0
○	93¢	1	1
○	I don't know.	4†	2†
C.	If you pay $3000 in taxes, about how much of your tax money will be spent on Agriculture?		
●	$ 210	38	52
○	$1305	9	4
○	$1410	6	2
○	$2100	11	12
○	Not enough information given	25	22
○	I don't know.	11	7†

†Figures do not total 100% because of rounding and/or nonresponse.

EXHIBIT 31. Percentages of Responses on Line Graph Exercise, Age 17

	Percent Responding Age 17
A. What percent of the labor force was unemployed in July, 1974?	
○ 4.9	4
● 5.5	87
○ 7.5	2
○ 8.6	4
○ I don't know.	3
B. What was the change in percent unemployed from July, 1974 to July, 1976?	
○ 0.0	0
● 2.0	66
○ 3.1	9
○ 5.5	4
○ 7.5	15
○ I don't know.	6
C. If the labor force totaled 98 million in July, 1974, approximately how many employable Americans were out of work that month?	
○ 4.80 million	16
● 5.39 million	36
○ 7.35 million	8
○ 8.43 million	8
○ I don't know.	30†

†Figures do not total 100% because of rounding and/or nonresponse.

EXHIBIT 32. Percentages of Correct Responses on Exercises With the Identical Information Presented in Different Ways (Circle Graph and Pictograph), Ages 9 and 13

	Percent Correct			
	Circle Graph		Pictograph	
	Age 9	Age 13	Age 9	Age 13
Direct reading	59	91	30	67
Simple interpretation	75	93	83	96

performance on the direct-reading part of the pictograph exercise compared with that of the circle graph item. Reading the pictograph required an extra translation step.

The results on the simple interpretation part of the two exercises were not so discrepant. For the circle graph exercise, students merely had to identify the largest section of the circle. For the pictograph exercise, students had to identify the "category" containing the most symbols. In this case, it did not matter how many individuals each symbol stood for.

Two more exercises presented identical information in different ways — one using a bar graph and the other using a table. The results from the different parts of these exercises are shown in Exhibit 33; performance was higher on items referred to by the table.

There were three "variables" involved in this set of exercises. Thus, the bar graph included several sets of three bars each, and the bars within a set were of different colors representing different categories of objects. In order to read the graph, the student first had to find the appropriate set of bars along the horizontal axis, then find the proper bar to refer to, and finally read the correct value on the vertical axis. In the table, this complication was taken care of by the use of a heading and subheadings along one dimension. Finding the correct entry in the table corresponds to finding the correct bar in the bar graph. Thus, reading a bar graph requires the extra step of determining the correct value on the vertical axis.

Two other questions not represented in the exhibit referred to a table in which the integer entries actually stood for hundreds of individuals. The majority of incorrect answers selected by students on those items indicate that a large number of students (more than 60% of the 13-year-olds and more than half of the 17-year-olds) actually used the entries as they appeared in the table and failed to multiply by 100 despite the unreasonableness of their answers. On exercises in which the

EXHIBIT 33. Percentages of Correct Responses on Exercises With the Identical Information Presented in Different Ways (Bar Graph and Table), Ages 9, 13 and 17

	Percent Correct					
	Bar Graph			Table		
	Age 9	Age 13	Age 17	Age 9	Age 13	Age 17
Direct reading	54	86	90	72	92	93
Simple interpretation	24	55	69	37	68	84
Multiple readings and computation	29	82	91	54	87	95

actual table entries were called for, the performance of students was substantially higher.

Geometric Manipulations

Many mathematical tasks (e.g., constructions, graphing) have spatial components. Assessment exercises in the category of geometric manipulations deal with the topics of symmetry, visualization and geometric constructions.

Given five different figures, approximately four-fifths of 9- and 13-year-olds could identify the figure that could "be folded so that both sides match." Given the same figures, however, less than one-fifth of either age group correctly identified the figure that was "symmetric." On the latter exercise, approximately half of the students answered, "I don't know." These results are another indication of the effect of vocabulary on the performance of a task that a majority of students can actually do.

Exhibits 34 and 35 give the results on exercises testing visualization skills. On the second of those exercises (Exhibit 35), the performance of the three age groups was not very different. Most of the results on items testing visualization follow a pattern similar to that seen in Exhibit 34. On most exercises administered to all three ages, the greatest gap in performance occurred between ages 9 and 13. There were two other exercises for which this was true: one exercise required the students to visualize what a solid object would look like when certain edges were cut and the figure unfolded, and the other required students to identify the figure that was identical to a given plane figure, but in a different position on a grid.

Several geometric constructions requiring the use of a compass and straightedge are commonly taught in the schools. Such constructions would include bisecting line segments, bisecting angles, constructing perpendiculars to lines at particular points and constructing congruent angles. Exercises involving two of those constructions were included in the assessment. On the exercises requiring 17-year-olds to actually perform the construction using a compass and a straightedge, 21% succeeded on one task and 25% on the other. One of those exercises was administered to 13-year-olds, but less than 2% of that age group succeeded. On two multiple-choice exercises that required students to select the diagrams that illustrated particular constructions, the percentages of correct responses for 13-year-olds on both items were 29% and 14%, compared with 53% and 38% for 17-year-olds.

Algebraic Manipulations

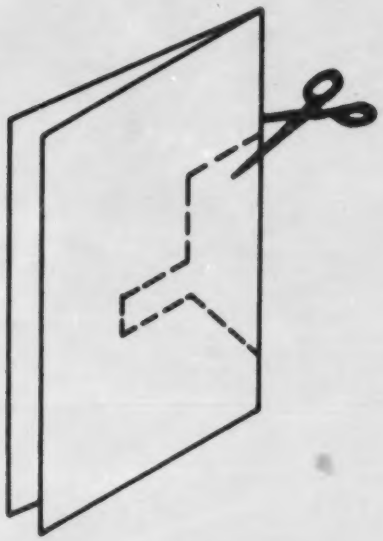
Although some students may not choose to take advanced mathematics, it has become clear in recent years that mathematics acts as a "critical filter" with respect to entrance into a variety of academic areas in college and, ultimately, into a large number of science-related and technological fields.

Approximately 72% of the 17-year-olds have taken at least a half year of Algebra I, and approximately 37% have taken at least a half year of Algebra II. The algebraic skills assessed by National Assessment are typical of those that would be taught in either Algebra I or Algebra II. They include simplifying algebraic expressions, substitution, solving equations and inequalities, and graphing. Most of the exercises in this section were administered only to 17-year-olds.

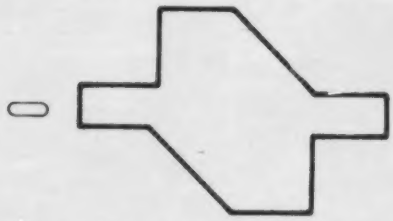
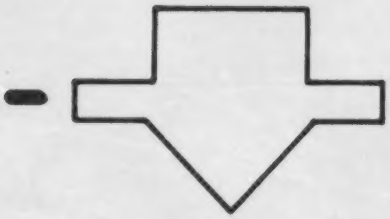
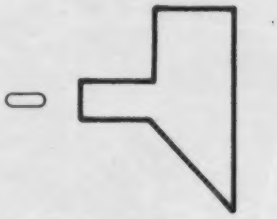
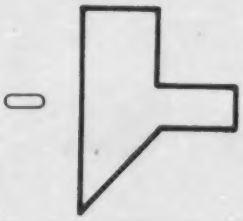
Exhibit 36 shows the results for several exercises involving the simplifying or factoring of algebraic expressions. The first four items show that between one-third and two-thirds of the 17-year-olds could simplify algebraic expressions that are not particularly complex. On the remaining exercises in the exhibit, requiring more complex algebraic manipulations, less than one-fourth of the 17-year-olds were successful.

A pair of related substitution exercises

EXHIBIT 34. Percentages of Responses on Symmetry Visualization Exercise, Ages 9, 13 and 17



What will the figure above look like when it is cut out and unfolded? Fill in the oval beside the one you choose.

	Percent Responding		
	Age 9†	Age 13	Age 17
	15	8	8
	60	80	86
	18	10	5
	5	2	1

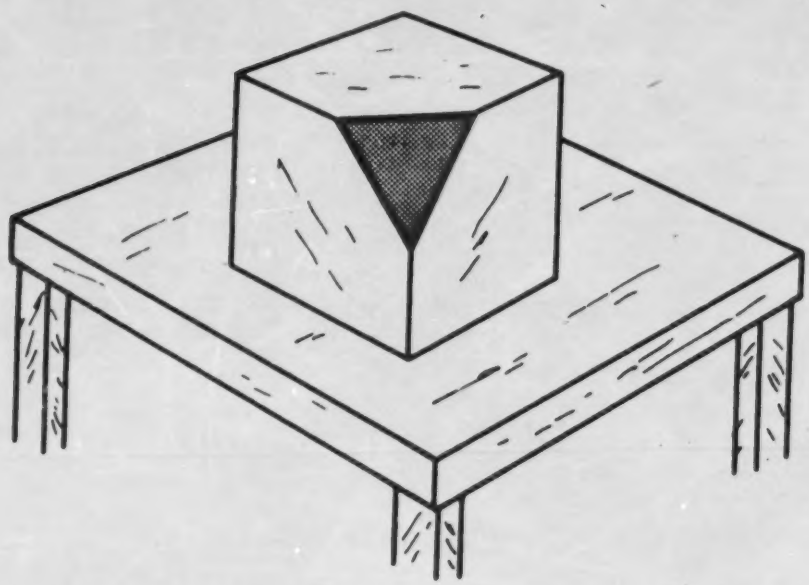
Percent Responding

I don't know. 1 *(Age 9)* 0 *(Age 13)* 0 *(Age 17)*

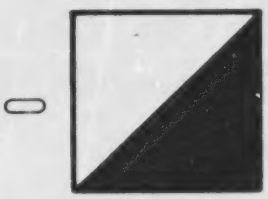
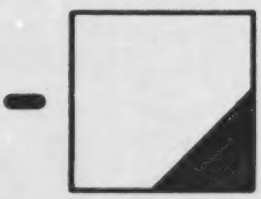
†Figures do not total 100% because of rounding and/or nonresponse.

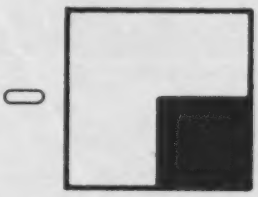
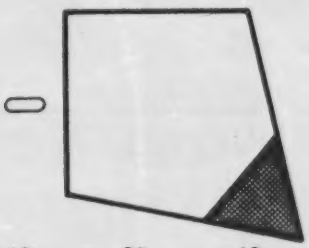
33

EXHIBIT 35. Percentages of Responses on Visualization Exercise, Ages 9, 13 and 17



The figure above shows a block with one corner cut off and shaded. Which of the following drawings shows how this block would look when viewed from directly above it? Fill in the oval beside the one you choose.

	Percent Responding		
	Age 9†	Age 13†	Age 17
	3	1	1
	70	76	79

	Percent Responding		
	Age 9†	Age 13†	Age 17
	4	3	1
	19	20	19

	Percent Responding		
I don't know.	3 *(Age 9)*	1 *(Age 13)*	0 *(Age 17)*

†Figures do not total 100% because of rounding and/or nonresponse.

EXHIBIT 36. Percentages of Correct Responses on Simplifying/Factoring of Algebraic Expressions Exercises, Age 17

	Percent Correct Age 17
Combining like terms in algebraic expression (multiple choice)	64
Using distributive property of multiplication over addition, then combining like terms (multiple choice)	52
Same as above except multiplication step involves negative integer multiplier (multiple choice)	34
Product of powers of x (multiple choice)	56
Power of x divided by power of x yielding negative power of x (multiple choice)	22
$3/2a + 3c/4b =$ (combining fractions – multiple choice)	14
Multiplying two binomials of the form $ax + b$ (open-ended)	26
Factoring quadratic expression – terms containing common factors (open-ended)	20
Factoring quadratic expression – sum and difference of squares (open-ended)	22
Factoring $x^2 - 5x + 6$ (open-ended)	22
Similar to above except coefficient of x^2 term greater than 1	16

showed the difficulty students had with functional notation. One exercise defined a simple linear function, f(x), and asked students for the value of f(a) where "a" was a particular small whole number. Twenty-eight percent of the 13-year-olds and 42% of the 17-year-olds succeeded. The other exercise asked essentially the same question except it did not use the functional notation f(x) or f(a). Seventy percent of the 13-year-olds and 90% of the 17-year-olds succeeded on this task, indicating again that performance is considerably higher when technical vocabulary is not involved.

On another exercise, 37% of the 17-year-olds correctly selected $12y^2 + 4$ as the value of $3x^2 + 4$ when $x = 2y$. Forty-two percent incorrectly selected $6y^2 + 4$. Given a very simple linear equation in slope-intercept form, a fifth of the students at age 17 successfully completed a table consisting of ordered pairs in which values of both x and y were missing.

Several exercises involved solving equations. Some of these exercises, especially those administered to 9-year-olds, did not require formal algebraic solutions. They presented the students with number sentences containing missing addends or factors and asked the students to find the missing values. On two missing-value exercises involving basic facts, performance was consistent with that on the basic fact exercises described in Chapter 1. Ninety-three percent of the 9-year-olds supplied the correct missing addend for one of those exercises, and on the other, 66% of the 9-year-olds and 91% of the 13-year-olds provided the correct missing factor. On the exercises that could not be answered quickly by inspection, performance dropped considerably. Fifteen percent of 9-year-olds and 54% of 13-year-olds named the number that would go in the box to make the number sentence □ - 19 = 32 true. Since Chapter 1 showed that most students could perform the computations that would be required to answer these exercises, it appears that many students are not aware of the relationship between inverse operations and number sentences in the exercises. About one-third of the 9-year-olds and three-fourths of the 13-year-olds correctly identified 1810 - 738 = □ as the number sentence that could be used to find what number belongs in the box in the number sentence □ + 738 = 1810. On a similar exercise, 24%, 75% and 86% of students at ages 9, 13 and 17, respectively, correctly identified the number sentence that could be used to find the missing factor in a multiplication sentence.

The variations of equations students may be asked to solve are numerous. For that reason, this report discusses the results of only a few specific examples. Thirty-two percent of the 17-year-olds solved an equation of the form $ax = b$ for x where a and b were decimal fractions; however, on a related computational exercise, 52% of that age group

accurately performed the division required for the solution of that equation.

Thirty-three percent of the 17-year-olds solved $30 = 2/5c + 10$, and 34% successfully found the value of x in $3x + 6 - 14 = x + 2$. Thus, about a third of the 17-year-olds successfully solved the simple linear equations included in the assessment exercises.

The quadratic equations $x^2 - 5x + 6 = 0$ and $(x - 2)^2 = 9$ were solved correctly by 18% and 8% of the 17-year-olds, respectively. Given the quadratic formula, 7% of the 17-year-olds solved a quadratic equation presented in standard form. On related knowledge exercises, less than half of the 17-year-olds identified a quadratic equation from a list of higher- and lower-order equations, and 23% identified the quadratic formula that could be used to solve a given quadratic equation. Twenty-eight percent and 44%, respectively, responded "I don't know" to these two knowledge exercises.

Solutions to the system of equations $n = 3k$ and $n + k = 72$ were found by about one-fifth of the 17-year-olds. Another system of equations in two unknowns contained equations of parallel lines. Just under half of the 17-year-olds knew that there was no pair of values that satisfied those equations. On another exercise, about the same number of students knew that the two lines described by those equations were parallel.

In addition to equations, students were also asked to solve inequalities, some of which could be solved as if they were equalities and some of which could not. Exhibit 37 shows the results on an exercise requiring students to identify the solution to a simple inequality. One-fourth and two-thirds of the students at ages 9 and 13, respectively, were successful.

Other exercises administered only to 13-year-olds asked them to select their solutions to two inequalities from the solution set $\{1, 3, 5, 7\}$. For the inequality $3 + \square > 5 + 1$, 48% correctly chose the answer 5 or 7. Fifty-four percent recognized that none of the elements in the solution set were solutions to $3 + \square > 5 + 5$.

Twenty percent of 17-year-olds selected $x > -4$ as being equivalent to $5 - 2x < 13$. Fifty-eight percent chose the correct equivalent inequality in a similar exercise, but that exercise did not require division by a negative integer that reverses the direction of the inequality in the first exercise.

An important prerequisite for a great deal of work in algebra is skill in identifying and locating points in one- and two-dimensional graphs. Graphs are the medium through which many algebraic concepts are understood. Fifty-eight percent of 13-year-olds correctly marked the point on the number line where one-half is. On a number line marked off in eighths of units, 43% of the students at age 13 correctly marked the location of one and three-fourths. Exhibit 38 shows the results on two exercises involving decimal fractions; the greatest "growth" with respect to plotting and identifying points occurs between the ages of 9 and 13.

Given a number line marked off in tenths of units and a point that was marked between 3.2 and 3.3, 34% of the 13-year-olds and 64% of the 17-year-olds named the number that represents the given point. Forty-six percent of the 13-year-olds and 69% of the 17-year-olds correctly marked the point 1.24 on a similar number line. Sixty-three percent of the 13-year-olds and 89% of the 17-year-olds correctly labeled a point corresponding to a negative integer on a number line on which only zero and two positive integers were labeled. Given another number line on which only two negative integers and zero were labeled, 72% of the 13-year-olds and 91% of the 17-year-olds correctly labeled a positive integer.

Maps are simple cases of graphs in two-dimensional form. Asked to identify the city on a map specified by given coordinates consisting of a letter and a number, 80%, 94% and 99% of students at ages 9, 13 and 17, respectively, succeeded.

EXHIBIT 37. Percentages of Responses on Solving Inequalities Exercise, Ages 9 and 13

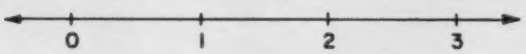
$5 + \square < 12$

Which number(s) below can go in the $\square$ to make this number sentence TRUE?

		Percent Responding	
		Age 9	Age 13
○	7	54	20
●	Any number less than 7	26	66
○	Any number greater than 7	12	11
○	I don't know.	5†	2†

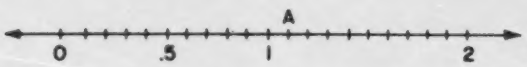
†Figures do not total 100% because of rounding and/or nonresponse.

EXHIBIT 38. Percentages of Responses on Points on Number Line Exercises, Ages 9, 13 and 17



Mark an X on the number line where 1.5 should be.

Percent Correct		
Age 9	Age 13	Age 17
33	78	88



What is the number at point A?

		Percent Responding		
		Age 9	Age 13	Age 17
○	.11	11	5	2
●	1.1	46	80	87
○	1.5	14	8	6
○	11	7	2	0
○	I don't know.	16†	3†	2†

†Figures do not total 100% because of rounding and/or nonresponse.

Two exercises required the students to select the ordered pair of numbers corresponding to points on a two-dimensional grid system. For a point in the first quadrant (only the first quadrant was depicted in this exercise), 44% of the 9-year-olds and 72% of the 13-year-olds identified the corresponding ordered pair. On a second exercise, 45% of the 13-year-olds and 78% of the 17-year-olds chose the correct ordered pair for a point in the third quadrant. The majority of students who did not select the correct options chose ordered pairs in which the ordinates and the abscissas were in the reverse order from the correct answers. On a slightly different task in which students were given the coordinates of a point in the fourth quadrant and asked to mark the point on a grid system, 39% of the 13-year-olds and 74% of the 17-year-olds were successful.

A great deal of attention in both algebra and geometry classes is devoted to graphing lines. Given a coordinate set of axes and a ruler, 18% of the 17-year-olds correctly drew the graph of $y = 2x + 1$, and 7% correctly graphed the inequality $y \leq 2x + 1$. Four percent of the 17-year-olds wrote the correct equation of a line when they were given the x- and y-intercepts. Fifteen percent of the 17-year-olds named the slope of the line $2y = 5x - 8$, and 11% named the y-intercept of the same line. Given the graph of a circle centered at the origin with a radius of six units, 3% of the 17-year-olds wrote the corresponding equation.

One would expect performance on algebraic manipulations to be more dependent on participation in particular mathematics courses than performance with respect to other mathematical topics. Five groups of 17-year-olds were defined on the basis of course backgrounds (see Appendix A). The average percentages of correct responses on algebraic manipulation exercises for the group that had taken the least advanced courses and the group that had taken the most advanced courses differed by 45 percentage points. Students who had taken the least advanced mathematics were defined as those who had

not taken Algebra I or any more advanced mathematics, while the students in the most advanced course-taking group had taken Algebra II and at least one more advanced course. One must be cautious, however, in attributing such a large difference to course training. The performance of the two groups differed almost as much in areas one would not expect to be so course dependent. For example, the difference between the two groups was 40 percentage points for computation exercises. Factors other than course background must also be taken into account.

Estimation Skills

Considering the increased availability of hand calculators and the fact that it is easy to make gross errors with these instruments by accidentally pressing wrong buttons, estimation skills seem to take on even greater importance than ever. Of course, estimation skills are constantly being used in everyday situations, such as deciding on tips, purchasing materials for home improvements, and so on. Assessment exercises dealt with estimating both computations and measurements.

Exhibit 39 shows the results on an exercise calling for an estimation of a missing addend. Thirty percent, 17% and 7% of the students at ages 9, 13 and 17, respectively, selected the answer 3000, indicating that they ignored the hundreds digit in the number 3900 when rounding to the nearest thousand. Nine-year-olds may have trouble with this skill since results on an unreleased exercise show that only 10% of the 9-year-olds correctly rounded a four-digit number to the nearest hundred. In estimating the answer to a subtraction problem involving four-digit numbers, 17%, 52% and 69% of students at ages 9, 13 and 17, respectively, succeeded. The wrong option chosen most frequently by all three age groups indicates that the students who chose that option failed to round to the nearest thousand when the digit in the hundreds place was greater than five.

Exhibit 40 depicts the results for estimations of computations involving ordinary fractions and mixed numerals. The percentages of 13- and 17-year-olds selecting the correct answers for these two exercises were quite low.

EXHIBIT 39. Percentages of Responses on Estimating a Missing Addend Exercise, Ages 9, 13 and 17

$3900 + \square = 6000$

Which one of the following is CLOSEST to the number that goes in the box?

		Percent Responding		
		Age 9	Age 13	Age 17
○	1000	5	3	2
●	2000	24	66	86
○	3000	30	17	7
○	5000	31	10	4
○	I don't know.	9†	3†	1

†Figures do not total 100% because of rounding and/or nonresponse.

Each item in another set of exercises asked students to select the closest estimate for the sum of several decimal fractions. The percentages of 13-year-olds answering these questions correctly were between 12% and 30%, while the 17-year-olds' percentages were between 33% and 52%. In some cases, one or more of the addends was completely insignificant in that the addends differed by orders of magnitude and some could be ignored entirely. These exercises appeared especially difficult; large numbers of 13- and 17-year-olds selected options indicating they tended to ignore the decimal point. Estimation skills related to multiplication and division of decimal numerals also appear to be lacking. Exhibit 41 depicts performance of 13- and 17-year-olds estimating quotients. Again, the percentages of correct responses were low.

Some assessment exercises required students to estimate measurements. Many of these items will not be released until after the

EXHIBIT 40. Percentages of Responses on Estimating Computations Involving Fractions Exercise, Ages 13 and 17

	Percent Responding Age 13	Age 17
A. ESTIMATE the answer to 12/13 + 7/8. You will not have time to solve the problem using paper and pencil.		
○ 1	7	8
● 2	24	37
○ 19	28	21
○ 21	26	15
○ I don't know.	14†	16†
B. ESTIMATE the answer to 5-3/7 + 6-5/8. You will not have time to solve the problem using paper and pencil.		
○ 8	4	2
○ 11	16	9
● 12	45	56
○ 15	23	18
○ I don't know.	11†	13†

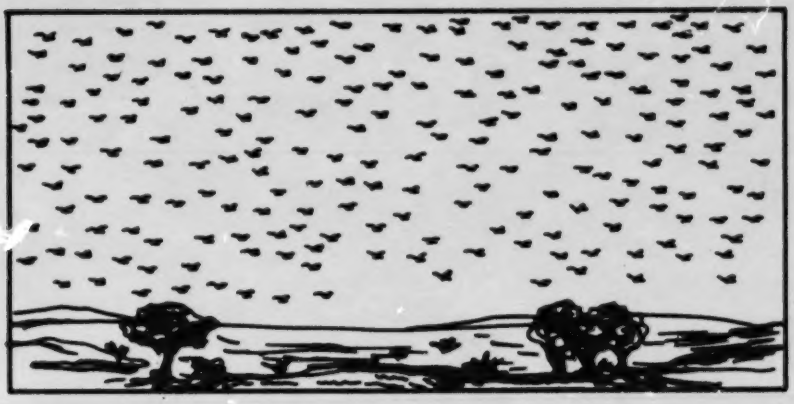
†Figures do not total 100% because of rounding and/or nonresponse.

EXHIBIT 41. Percentages of Responses on Estimating Computations Involving Division of Decimal Fractions Exercise, Ages 13 and 17

	Percent Responding Age 13	Age 17
30 ÷ 317		
● .1	7	15
○ .01	22	30
○ 1	6	8
○ 10	59	41
○ I don't know.	5†	6
250 ÷ .5		
○ 50	61	47
● 500	25	39
○ 1000	3	3
○ 1250	4	6
○ I don't know.	6†	4†
.239 ÷ .4		
○ .006	24	20
○ .06	34	32
● .6	19	28
○ 6	8	7
○ I don't know.	13†	12†

†Figures do not total 100% because of rounding and/or nonresponse.

EXHIBIT 42. Percentages of Responses on Estimating a Large Number of Objects Exercise, Age 9



This is a picture of birds flying south for the winter. About how many birds are in the picture? You will not have time to count them.

	Percent Responding Age 9
○ 20	1
● 200	73
○ 2000	22
○ I don't know.	4

next mathematics assessment. Exhibit 42 shows an exercise for 9-year-olds requiring them to estimate a large number of objects. While almost three-fourths of the respondents selected the correct response, more than a fifth selected the response that was 10 times more than the correct one. This result probably reflects the inexperience of many 9-year-olds with both large numbers and estimation.

Some of the other estimation exercises show that students were relatively unfamiliar with metric units, as opposed to English units, of measure. On an exercise requiring students to select the closest estimate of the height of a common object expressed in English units, 54% of 9-year-olds, 79% of 13-year-olds and 86% of 17-year-olds selected the correct measurement. However, substantially fewer respondents in all three age groups (20%, 37% and 50%, respectively) selected the correct estimate of the height of the same object expressed in metric units.

CHAPTER 4

GROUP RESULTS AND AGE-LEVEL COMPARISONS

To examine the relationships between the characteristics of students and mathematics achievement, National Assessment analyzed results for various groups within the national population. Some of these groups are defined by students' backgrounds or personal characteristics; others are related to school experiences. In addition to discussions of various groups' results at each age level, this chapter makes comparisons between age groups based on performance on identical sets of mathematical knowledge and skill items administered to more than one age group.

Group Results

Groups related to students' background characteristics were based on the following factors: region of the country, sex, race, level of parents' education, and type and size of community lived in. Groups based on school experience were defined by: grade level in school, and, for 17-year-olds, types of mathematics courses taken. Definitions of groups reported upon appear in Appendix A.

National Assessment found that similar patterns of group results on the knowledge and skills items prevailed across age groups — the same groups were generally either above or below national levels, although in some cases tendencies became more marked for the older students.

Description of the Data

Group performance is described in terms of average performance on a particular set of exercises and is compared with the national performance level on that item set. Average performances have been computed for all knowledge items, for all skill items and for the various subsets of those items. Exhibits 43—49 and 51 display differences between group and national performance for the various types of groups on the entire sets of mathematics knowledge and skill items. In these exhibits, the zero line represents the national performance level. The solid line for each group represents the difference between that group's mean and the national mean. A positive difference means that the group performed above the nation, and a negative difference, below it. The bar surrounding each mean represents two standard errors above and below the mean achievement for the group.

A standard error of the sample mean is an estimate of the sampling variability among the means of all possible samples; it is used to estimate the precision of the mean obtained in a particular sample. Intervals of two standard errors below to two standard errors above a particular statistic would include the average of the statistic in approximately 95% of all possible samples of the same size. An interval computed in this way is a 95% confidence interval. If the standard error bar for a particular group mean does not cross the national line, one can be reasonably sure that the group mean is different from the national mean.

It should be remembered that differences in achievement levels cannot be ascribed solely to membership in a particular group being discussed. Research has shown that any number of socioeconomic, school-related and environmental factors contribute to achieve-

ment, and caution must be taken not to attribute a cause-and-effect relationship between membership in a specific group and performance.

Background Variables

The following sections describe patterns of results for the various groups. Appendix B shows group performance relative to the national level for all knowledge and skill items and for various subsets of those items. The patterns of group differences were generally consistent across ages and across the two cognitive levels — mathematical knowledge and skill. That is to say, groups that tended to perform above or below the national average on knowledge exercises at one age tended to do so at the other ages as well, and the same patterns were usually seen for skill exercises. Exhibits 43—49 and 51 show the relative performance of the various groups of students. When particular subtopics of mathematical knowledge and skill were examined separately, the differences between groups were still consistent with the more general trends seen for all knowledge or skill exercises with few exceptions. The general patterns of group differences and some exceptions to those patterns are described in the following sections.

Region of the country. As seen in Exhibit 43, the relative performance of the regions was fairly constant across age levels for both knowledge and skill exercises. The Northeastern (NE) and Central (C) regions were above the nation, the Southeast (SE) was below the nation and the West (W) was generally below it. The patterns depicted in Exhibit 43 were evident for most of the subsets of knowledge and skills items with the exception of whole number computations, for which regional differences were practically nonexistent at age 17.

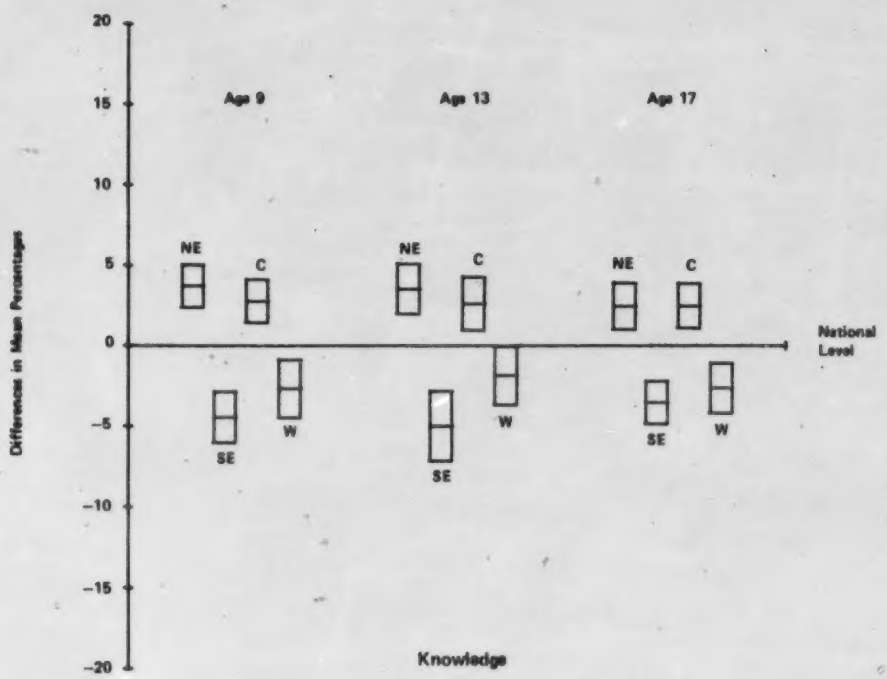
Sex. Unlike differences between other types of groups discussed, differences in males' (M) and females' (F) performance were not constant across ages (Exhibit 44). On knowledge exercises, females outperformed males at age 9; there was no difference in performance at age 13; but by age 17, males outperformed females. On skill items, there was little difference in performance at age 9, but females had the advantage at age 13, while 17-year-old males outperformed 17-year-old females.

Patterns of overall performance as seen in the exhibits can be somewhat misleading, since they do not hold for several subsets of the knowledge and skill exercises. For example, although there was little difference between the sexes in overall skills performance at age 9, females outperformed males by more than 5 percentage points on whole number computation, while males did better than females by about 4 percentage points on estimation skills. At this age, males were slightly better on measurement skills than females.

At age 13, males outperformed females on knowledge of measurement by 7 percentage points, although there was no difference on knowledge items overall. Thirteen-year-old females were almost 8 percentage points higher than their male counterparts on decimal computations, while males outperformed females by 5 percentage points on measurement skills.

Differences in performance were greatest at age 17; males were about 2 and 2½ percentage points above females for all knowledge and for all skills items, respectively. However, differences between male and female 17-year-olds were negligible on knowledge of basic computation facts and properties of numbers and operations and on skill in computation with whole numbers, fractions and decimals. However, males at this age had sizable advantages in some areas. Males outperformed females in knowledge and skill items concerning measurement by about 7 and 10 percentage points, respectively. Measurement appears to be an area in which males have an advantage regardless of age level. Seventeen-year-old males' knowledge of number order and skills in geometric manipulations and estimation surpassed those of females by 6 percentage points.

EXHIBIT 43. Differences in Mean Percentages Between Regional Groups and the Nation on Knowledge and Skills Items, Ages 9, 13 and 17



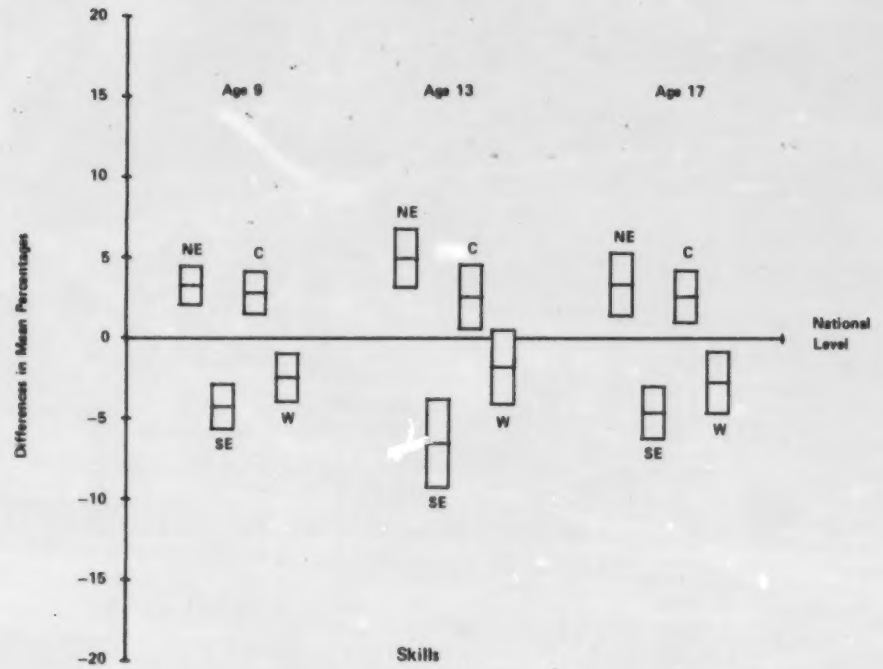
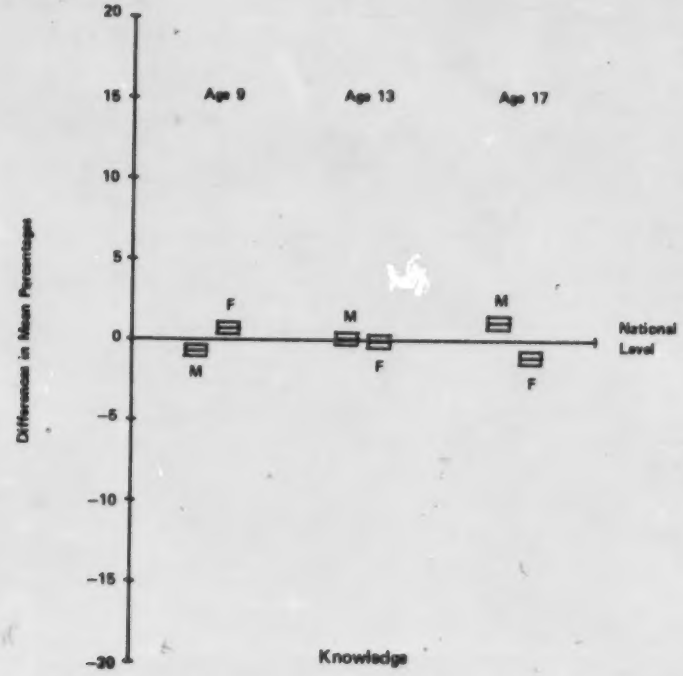
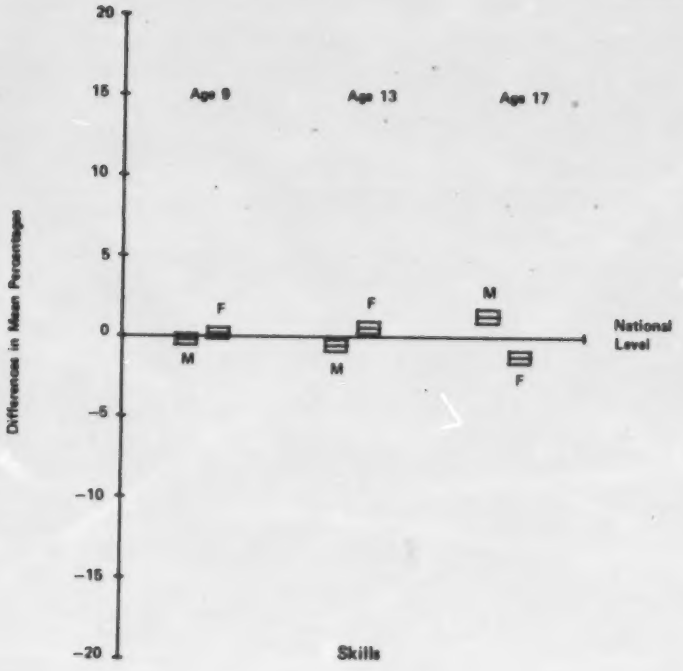


EXHIBIT 44. Differences in Mean Percentages Between the Sexes and the Nation on Knowledge and Skills Items, Ages 9, 13 and 17





Different course backgrounds of males and females have been cited as a possible cause of the differences in mathematical performance. Forthcoming National Assessment documents will report on the extent to which course enrollment patterns can account for performance differences. In recent years, researchers have referred to a 2:1 ratio of males to females enrolled in the more advanced mathematics courses. The 1977—78 assessment found a 3:2 ratio of 17-year-old males to 17-year-old females who had taken advanced mathematics courses beyond Algebra II. Also, the percentage of the total student population taking such courses was not very large. For example, 5% of the 17-year-old males had taken Calculus or Pre-calculus, compared with 3% of the females.

Sex differences in enrollments were not found for lower-level courses including Algebra I, Geometry and Algebra II. It may be that differences in the mathematics enrollment patterns of the sexes are diminishing, and it is not likely that the differences that do exist can account for the total performance difference.

Race. The average achievement of whites (W) was higher than that of Hispanos (H) and blacks (B) at all three age levels on both knowledge and skill exercises. Hispanic achievement tended to be higher than that of blacks (Exhibit 45). The differences between the three racial groups reported upon appear to increase between the ages of 9 and 13. For example, the gaps between whites and blacks for mathematical knowledge were approximately 13, 17 and 18 percentage points at ages 9, 13 and 17, respectively. The corresponding percentage differences for skills were about 13, 20 and 20.

At age 9, differences between racial groups appear greatest on knowledge of place value, knowledge of basic computation facts and skill at reading graphs and tables. Racial differences at ages 13 and 17 were greatest on knowledge of place value, computations with decimals and measurement skills. Differences at age 13 were also large for computation with fractions. At age 17, performance of the different races was also quite disparate on knowledge of number order.

There were some important differences between blacks and whites with respect to mathematics course enrollment patterns, especially for lower-level academic mathematics courses. Three-fourths of 17-year-old whites had taken Algebra I, compared with 55% of 17-year-old blacks. The percentages of 17-year-old whites and blacks who had taken Geometry were 55% and 31%, respectively, and for those who had taken Algebra II, 38% and 24%, respectively.

Parental education. The relative position of the various parental-education groups remained constant at all three ages, with those whose parents had more education doing better than those whose parents had less education (Exhibit 46). For both mathematical knowledge and skill items and most subtopics, students whose parents had some education beyond high school (PH) performed 4 to 7 percentage points higher than the national mean percentage; those whose parents graduated from high school (GH) but did not continue their education performed near the national mean; and students whose parents did not graduate from high school (NG) scored 5 to 13 percentage points below the national mean. Differences between parental-education groups were greatest on reading graphs and tables for 9-year-olds, computations with fractions for 13-year-olds, and knowledge of place value and number order and skill at computing with integers at age 17.

Type of community. National Assessment reports upon three extreme types of communities: advantaged urban (AU), disadvantaged urban (DU) and extreme rural (ER). As shown in Exhibit 47, the advantaged-urban groups performed consistently above the national level on both knowledge and skill items, while the other two groups performed below the national level. There was a slight tendency for achievement of disadvantaged-urban students, relative to the nation, to

EXHIBIT 45. Differences in Mean Percentages Between Racial/Ethnic Groups and the Nation on Knowledge and Skills Items, Ages 9, 13 and 17

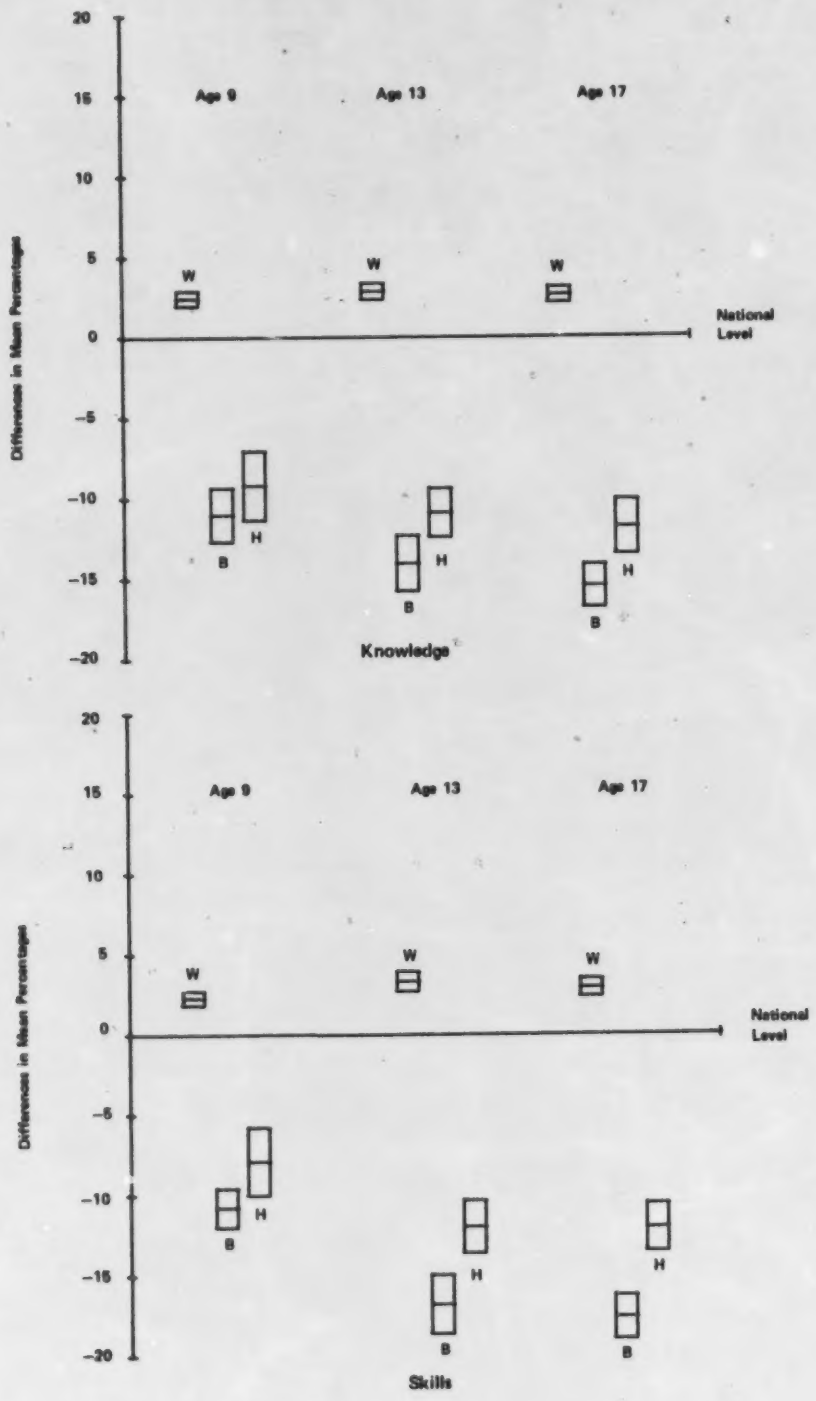
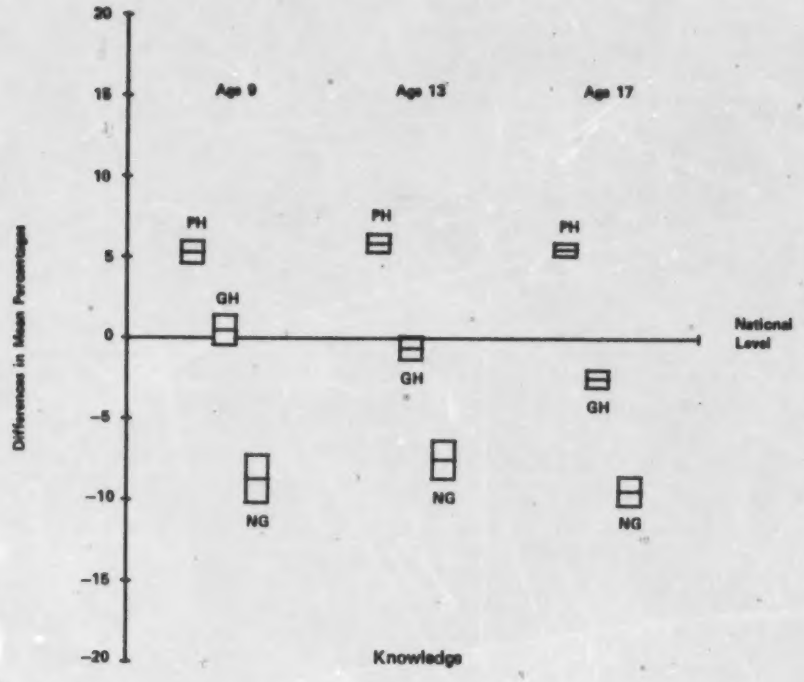


EXHIBIT 46. Differences in Mean Percentages Between Parental-Education Groups and the Nation on Knowledge and Skills Items, Ages 9, 13 and 17



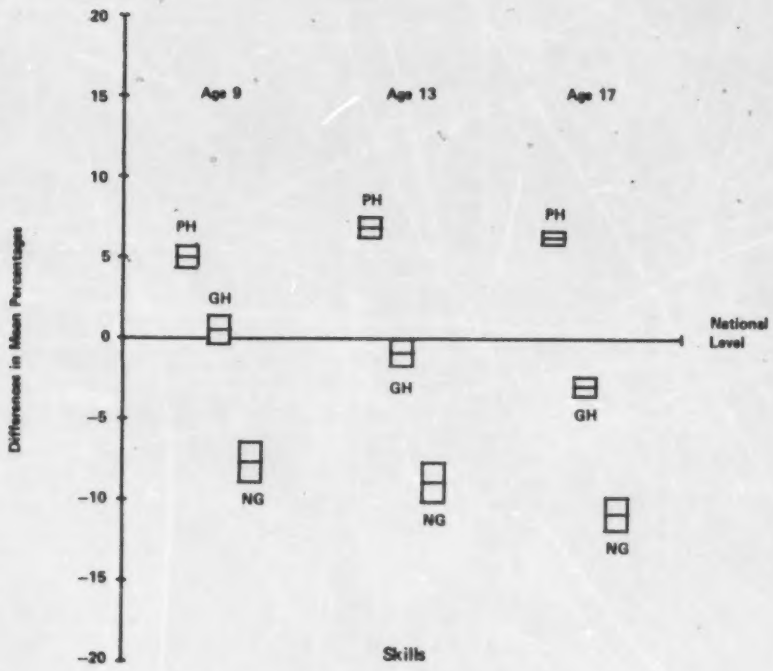


EXHIBIT 47. Differences in Mean Percentages Between Type-of-Community Groups and the Nation on Knowledge and Skills Items, Ages 9, 13 and 17

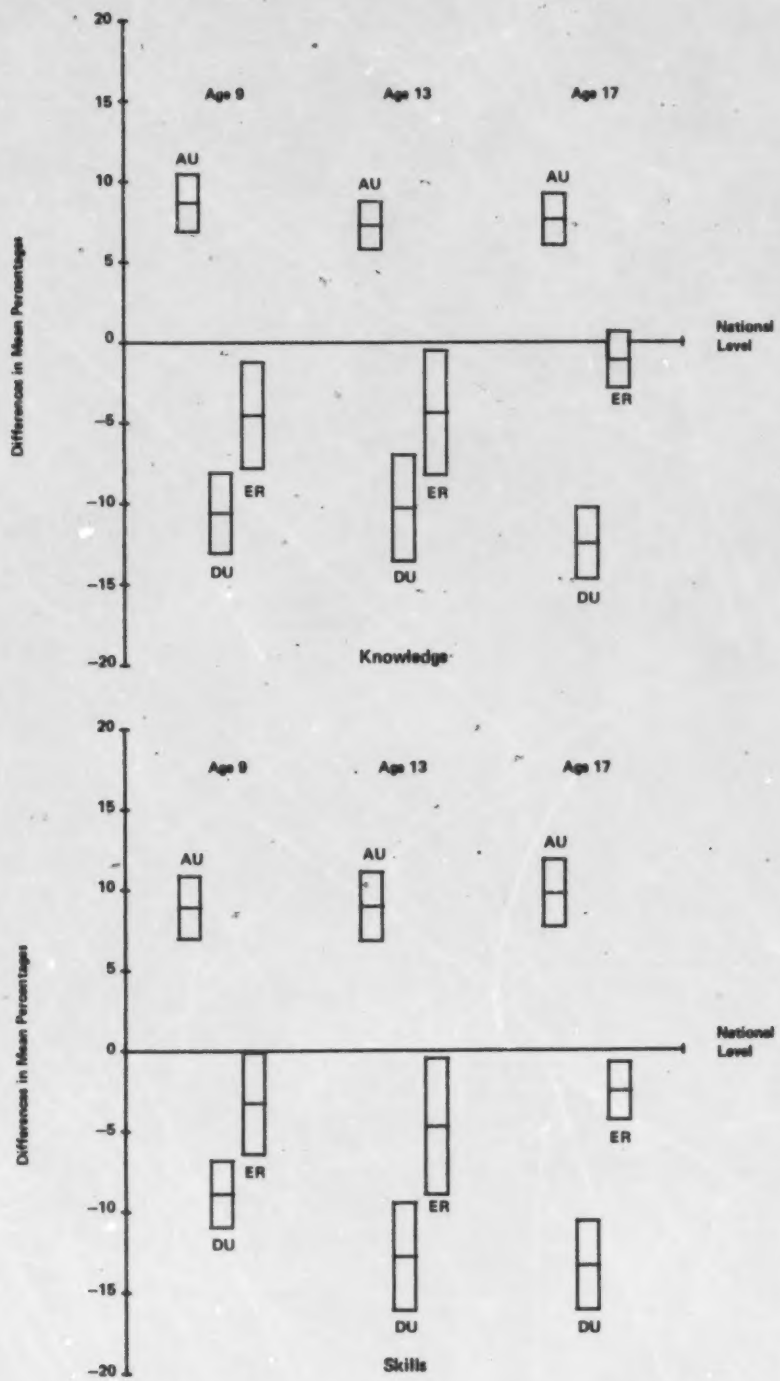
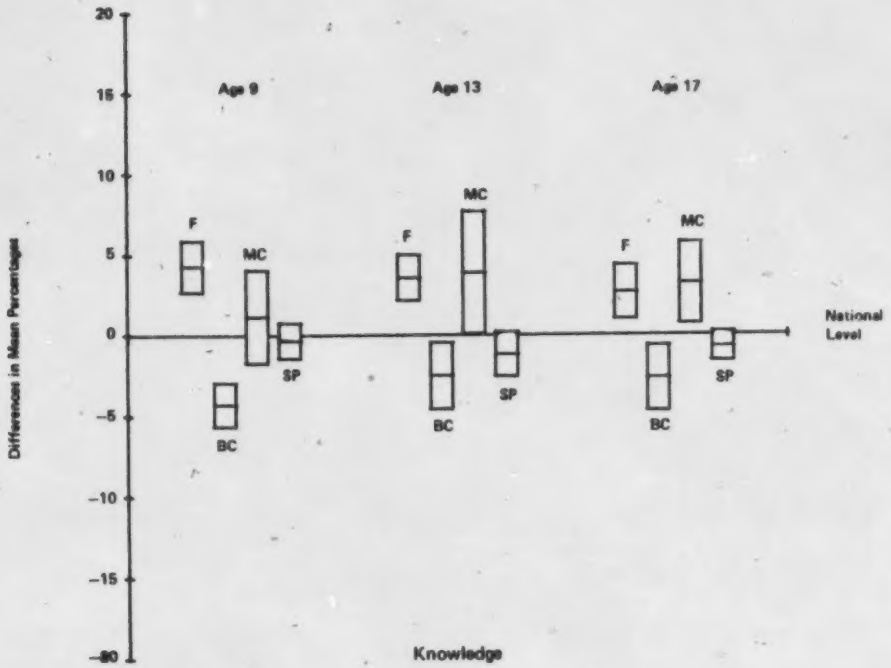
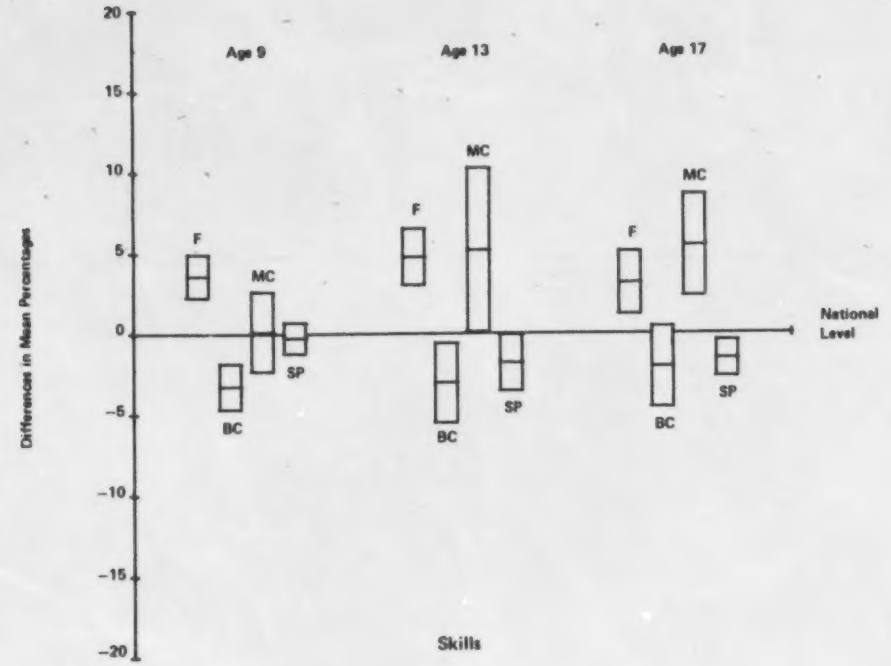


EXHIBIT 48. Differences in Mean Percentages Between Size-of-Community Groups and the Nation on Knowledge and Skills Items, Ages 9, 13 and 17

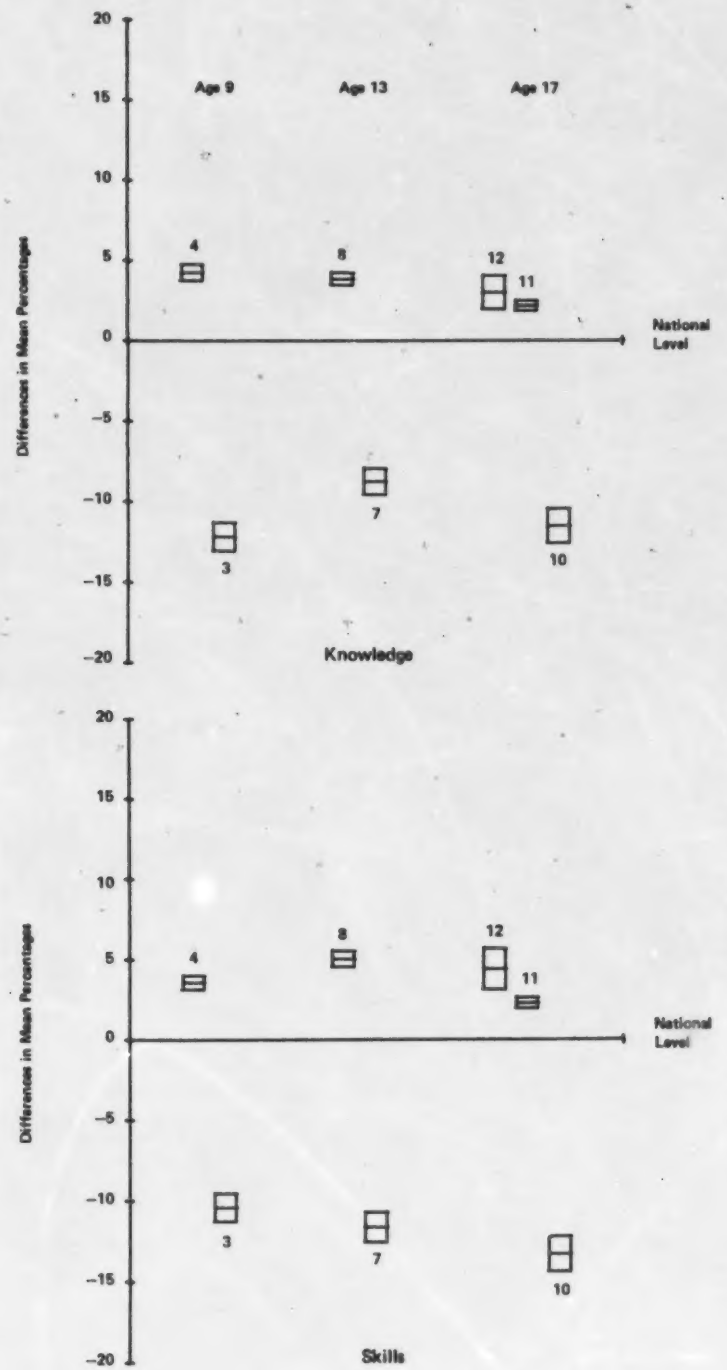


Knowledge



Skills

EXHIBIT 49. Differences in Mean Percentages Between Grade-in-School Groups and the Nation on Knowledge and Skills Items, Ages 9, 13 and 17



decline with age, while that of extreme-rural students tended to improve for 17-year-olds. The largest differences between advantaged-urban and disadvantaged-urban students were found in knowledge of basic computation facts, skill at whole number computation, and reading graphs and tables at age 9; in knowledge of place value and decimal computation skills at age 13; and knowledge of number order and integer computational skills at age 17.

Size of community. Four kinds of communities defined solely on the basis of size are fringes around big cities (F), big cities (BC), medium cities (MC) and smaller places (SP). The fringes around big cities were above the nation at each age, and big cities were below national levels (Exhibit 48). Nine-year-olds from medium cities and smaller places performed at the national level. At the older ages, however, medium cities tended to be above the nation.

School Variables

Grade in school. In the National Assessment sample, about three-fourths of the 9-year-olds are in the 4th grade (4) and one-fourth in the 3rd grade (3); about three-fourths of the 13-year-olds are in the 8th grade (8) and one-fourth in the 7th grade (7); and about 75% of the 17-year-olds are in the 11th grade (11), with about 15% in the 10th (10) and 10% in the 12th (12) grade. As one would expect, those in a higher grade at each age level showed higher mathematics achievement than those in a lower grade (Exhibit 49).

Mathematics courses taken (17-year-olds). Seventeen-year-olds were asked to supply information on mathematics courses they had taken. Exhibit 50 shows the percentages who reported that they had taken each of the various courses.

Mathematics achievement on knowledge and skill items was analyzed for the following groups of 17-year-olds: (1) those who had not taken any of the courses listed in Exhibit 50 (<Alg I); (2) those who had taken Algebra I but no more advanced courses (Alg I); (3) those who had taken Geometry but not Algebra II or any more advanced courses (Geom); (4) those who had taken Algebra II but not Trigonometry, Probability and Statistics, or Pre-calculus (Alg II); (5) those who had taken Algebra II and Trigonometry, or Probability and Statistics, or Pre-calculus (>Alg II).

EXHIBIT 50. Percentages of 17-Year-Olds Taking Various Mathematics Courses

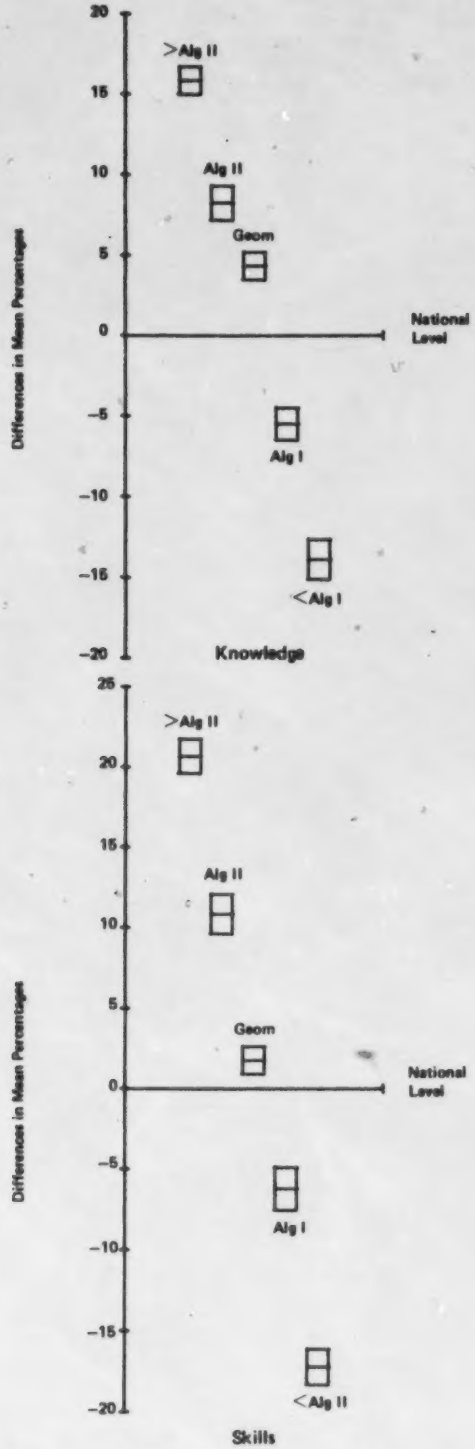
	Percent Who Reported They Had Taken Course for ½ or 1 Year
Algebra I	72
Geometry	51
Algebra II	37
Trigonometry	13
Probability and statistics	3
Pre-calculus	4

Exhibit 51 shows performance for the different levels of mathematics course-taking experience. Sizable differences (as much as 57 percentage points) were apparent for the different levels of mathematics courses taken on all subsets of knowledge and skill items analyzed. Differences appeared greatest on algebraic manipulations; computations with fractions and integers; and computing perimeter, area and volume. Of course, even though these topics are taught in the middle grades, one would expect experience in secondary mathematical courses (e.g., algebra and geometry) to be closely associated with performance in these areas.

Age-Level Comparisons

The achievement levels of the three age groups assessed by National Assessment are compared in this section. Previous compari-

EXHIBIT 51. Differences in Mean Percentages Between Groups Based on Mathematics-Course Background and the Nation on Knowledge and Skills Items, Age 17



sons discussed were made on the basis of all knowledge and skill items administered to a particular age group; comparisons in this section are based on only those items taken by more than one age group. Nine- and 13-year-olds responded to 78 identical knowledge and 98 identical skill exercises; 13- and 17-year-olds answered 120 knowledge and 218 skill exercises that were the same.

As one might expect, performance improved between ages 9 and 13 and between 13 and 17. The improvement was more pronounced between ages 9 and 13.

On knowledge items, 13-year-olds' mean performance level was 24 percentage points above that of 9-year-olds; while the performance of 17-year-olds was 12 percentage points higher than that of 13-year-olds. On skill exercises, the gap between 9- and 13-year-olds was 28 percentage points; while the gap between students at ages 13 and 17 was 17 percentage points.

Exhibits 52 through 55 compare the performance of selected reporting groups for two age levels given identical knowledge and skill items. As seen in Exhibits 52 and 53, 13-year-olds uniformly showed higher achievement in mathematical knowledge and skills than 9-year-olds. The same pattern did not hold for 13- and 17-year-olds (Exhibits 54 and 55). The assessment found that, on the average:

- A higher percentage of white 13-year-olds answered knowledge items correctly than did 17-year-old blacks and Hispanos.

- A higher percentage of white 13-year-olds answered skill items correctly than did 17-year-old blacks.

- Thirteen-year-olds in advantaged-urban communities did better than 17-year-olds in disadvantaged-urban communities on both knowledge and skill exercises.

- Thirteen-year-olds whose parents had some post-high-school education outperformed 17-year-olds whose parents did not graduate from high school on knowledge items. Those same two groups performed at about the same level on mathematical skill exercises.

The improvement between 9- and 13-year-olds was very similar for most reporting groups analyzed. However, 13-year-old blacks and Hispanos were farther below the national level than their 9-year-old counterparts; thus, improvement between ages 9 and 13 for these groups was less than the national improvement. Interestingly, on mathematical knowledge items, improvement between the ages of 9 and 13 was greater for students from disadvantaged-urban communities than for students from either advantaged-urban or extreme-rural communities.

Similarly, the improvement between ages 13 and 17 was very much the same for most reporting groups. However, on both knowledge and skill items, students from extreme-rural communities showed greater improvements between the ages of 13 and 17 than did the nation as a whole. Students from disadvantaged-urban communities improved somewhat less than students from either advantaged-urban or extreme-rural communities on knowledge exercises. This is the reverse of what was true for this group with respect to improvement between the ages of 9 and 13.

EXHIBIT 52. Average Percentages of Correct Response on Identical Sets of Knowledge Items for Selected Reporting Groups, Ages 9 and 13 (Number of Items = 78)

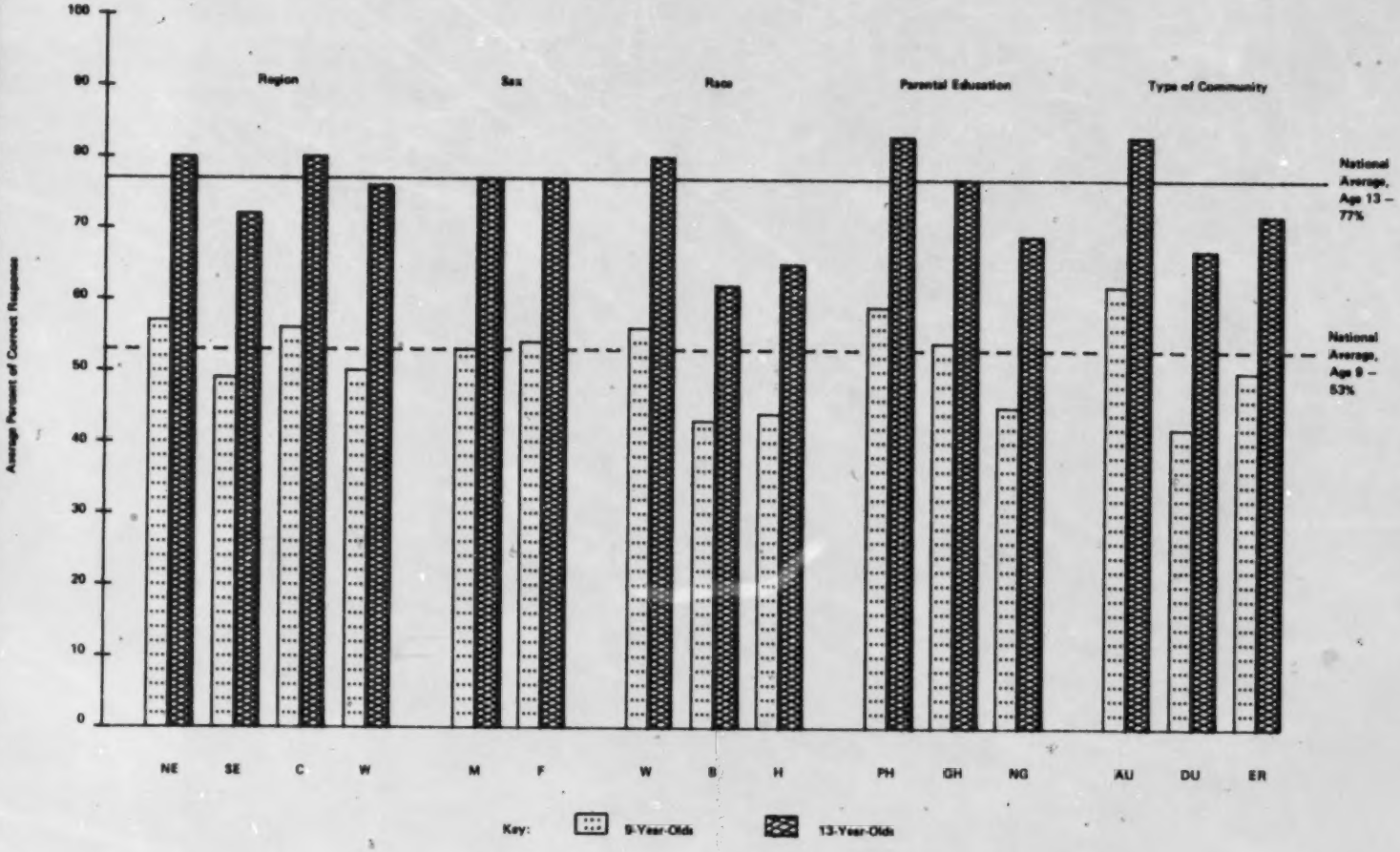


EXHIBIT 53. Average Percentages of Correct Response on Identical Sets of Skills Items for Selected Reporting Groups, Ages 9 and 13 (Number of Items = 98)

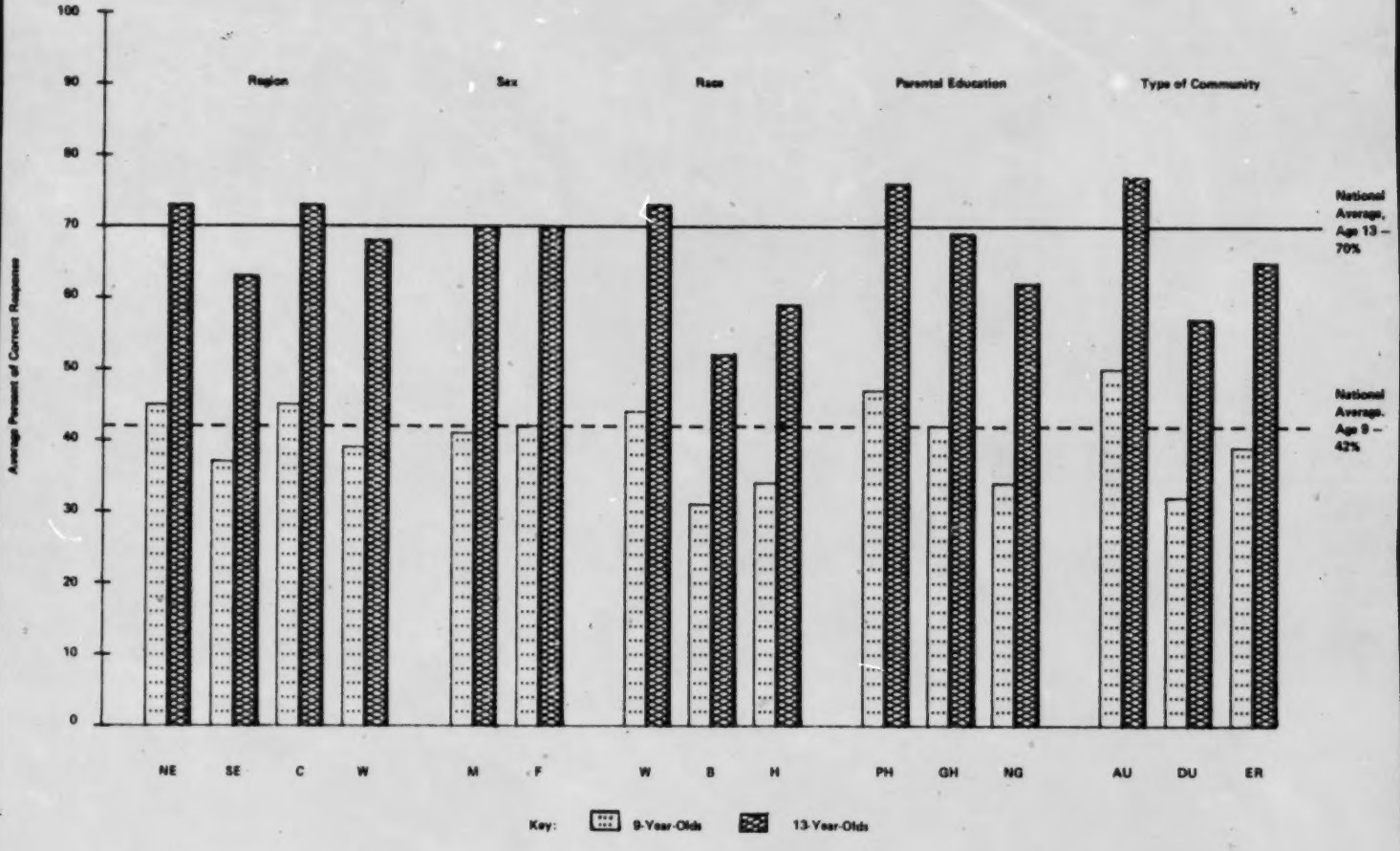


EXHIBIT 54. Average Percentages of Correct Response on Identical Sets of Knowledge Items for Selected Reporting Groups, Ages 13 and 17 (Number of Items = 120)

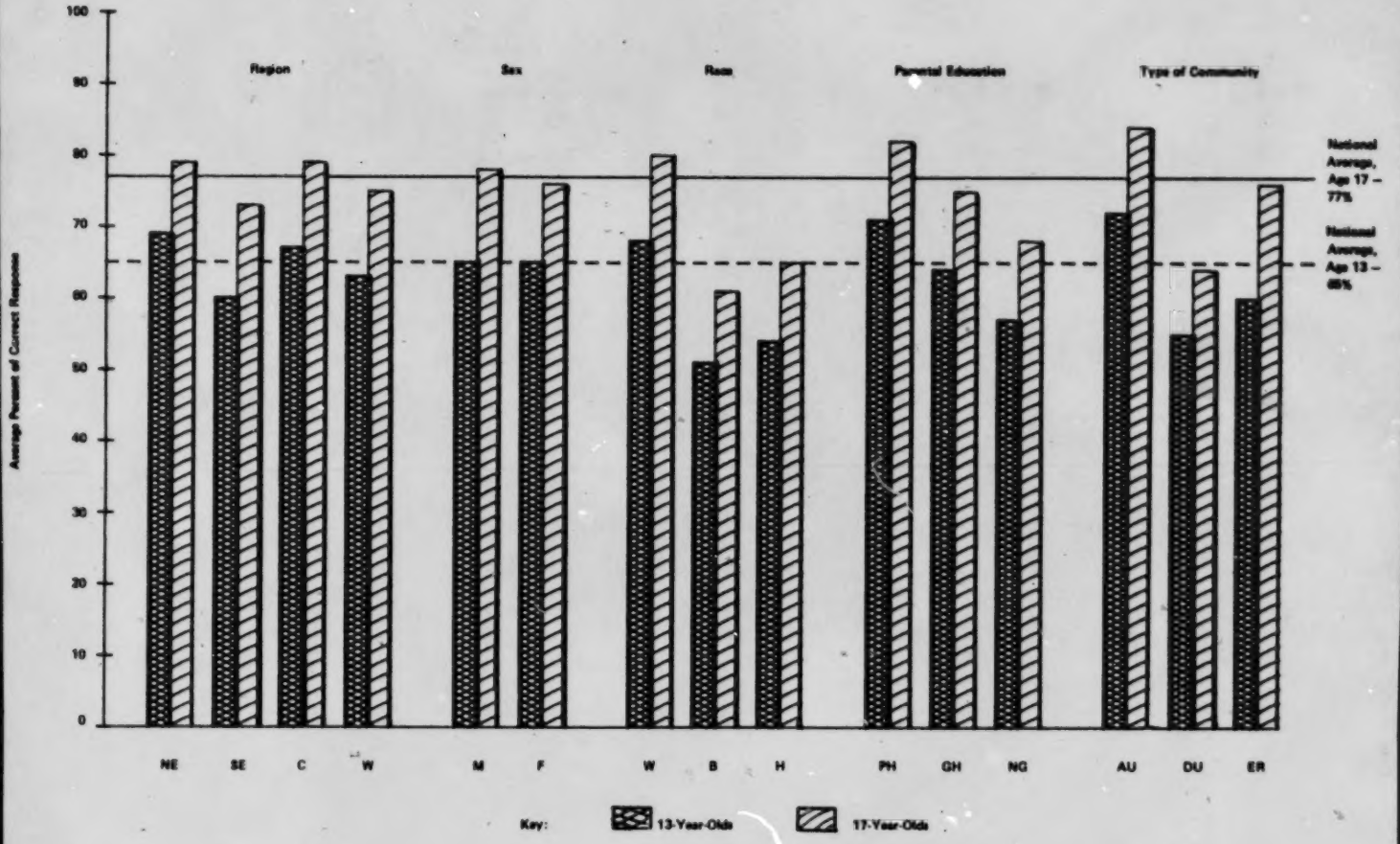
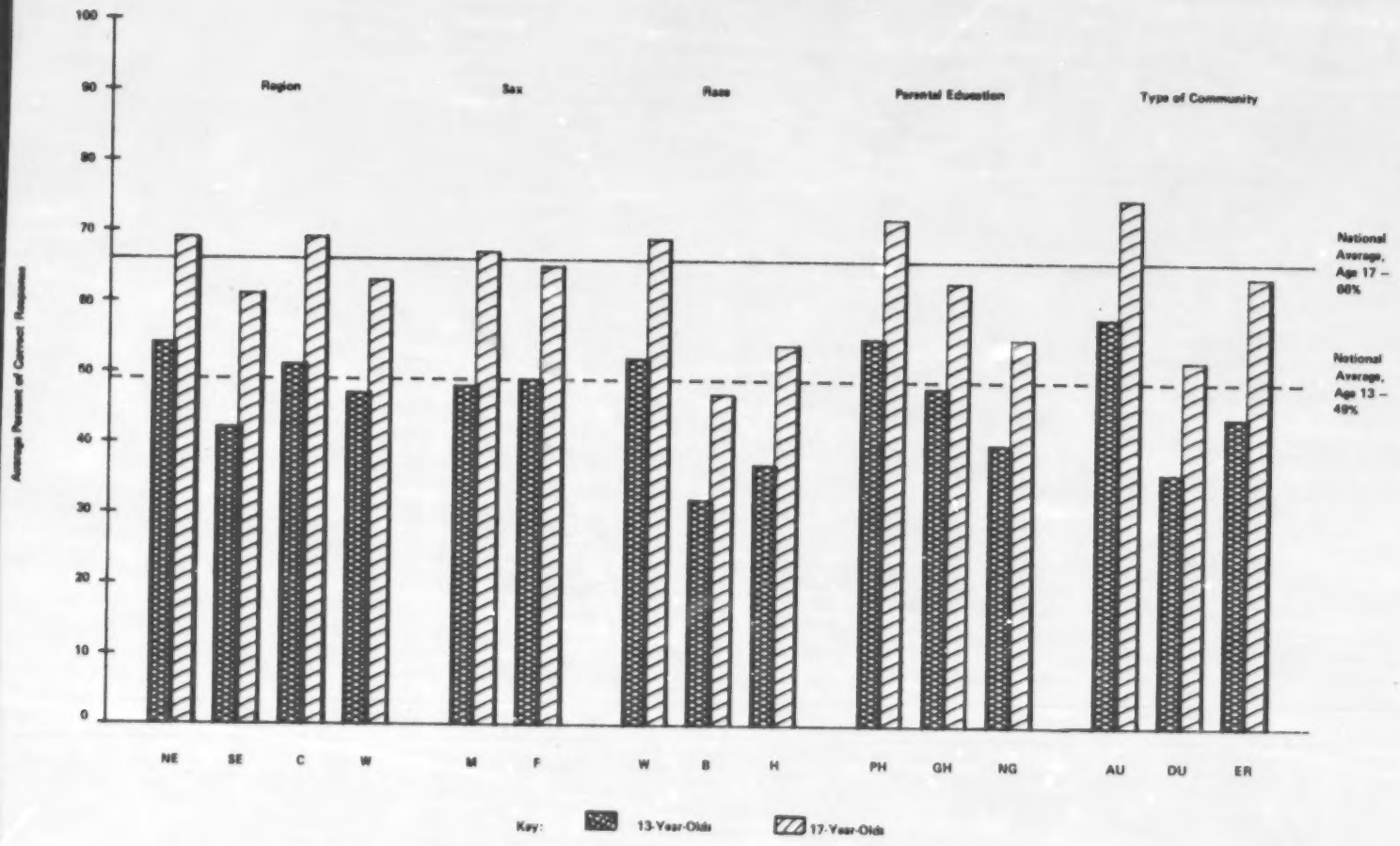


EXHIBIT 55. Average Percentages of Correct Response on Identical Sets of Skills Items for Selected Reporting Groups, Ages 13 and 17 (Number of Items = 218)



CHAPTER 5

OBSERVATIONS ON THE RESULTS

The 1977–78 national assessment of mathematics provides a comprehensive survey of students' present levels of mathematics achievement. The extensive data described in this report are best interpreted in light of current expectations and trends in mathematics education. Accordingly, National Assessment convened a panel of persons active in the field to interpret the results and to consider the implications for mathematics education.

Panel members included: Iris Carl, mathematics resource center specialist and curriculum writer, Houston (Texas) Independent School District, and member of the National Council of Teachers of Mathematics (NCTM) Program and Instructional Affairs Committees; Thomas Carpenter, associate professor of curriculum and instruction, University of Wisconsin, Madison; Robert Dilworth, professor of mathematics, California Institute of Technology, Pasadena; Edgar Edwards Jr., associate director of mathematics (K-12), Virginia Department of Education, Richmond, and member of the NCTM Board of Directors; and James W. Wilson, professor and department head, Mathematics Education Department, University of Georgia, Athens, and member of the NCTM Board of Directors.

This chapter presents selected quotes and a summary of comments made during a two and one-half day meeting at which the panel informally reviewed and discussed the mathematics results. Observations of the panel members represent their opinions as individuals and do not necessarily reflect the positions of any organizations with which they are associated.

The panel examined results for the mathematics cognitive process levels of knowledge, skills and application in addition to data on changes in performance between 1973 and 1978. The observations presented here, although focusing on the cognitive processes of knowledge and skill, also reflect data found in other NAEP reports (*Mathematical Applications*, 1979; *Changes in Mathematical Achievement, 1973–78*, 1979).

Panel members emphasized cautions that must be kept in mind when interpreting the data. The format of and the vocabulary used in an item can have an influence on performance. For example, multiple-choice items are often easier to answer than open-ended items, even if the task is the same. Specific vocabulary may present difficulties, as in the case in which many more students could calculate the "distance around" a figure than could find the "perimeter" of the same figure. Thus, in generalizing from performance on an item to ability to perform a task, one must take into account the nature of the particular items used in the assessment. In addition, although students are told that the assessment is not a test and that their scores will not be recorded, the assessment is a timed, test-like situation, and responses given under the stress of the situation may not be totally accurate indicators of ability.

A Historical Perspective

Amid reports of declining test scores and public disenchantment with curricular and teaching innovations, there has been widespread demand during the 1970s that educa-

tion go "back to the basics." A number of states and local districts have instituted "minimal-competency" requirements for high school graduates, and schools increasingly have been held "accountable" for the performance of their students.

In mathematics, the movement to the "basics" has often resulted in a narrowing of the curriculum, stressing computational skills and knowledge of facts and definitions with less time available for instruction in other topics. Wilson noted that textbooks have changed during the 1970s: "By the mid-1970s, textbooks in the schools emphasized computational skills. Word problems were streamlined to present students with the least possible amount of complexity."

The increased testing associated with minimal competency and accountability movements may encourage teachers to emphasize material found in tests and to devote less effort to material that is not generally tested. Teachers also may be inclined to stress areas that can be fairly easily impacted by increased classroom effort — namely lower-level cognitive processes such as knowledge and simple skills.

Training of mathematics teachers is not consistent across the country, and some persons teaching mathematics may not receive sufficient training. For example, elementary teachers may not be required to take any mathematics beyond what they have had in high school. This may lead to a fear of teaching mathematics. Dilworth described a group of elementary teachers at workshops held at the Teacher Center in San Diego, California: "Out of 65 teachers, many indicated that prior to the workshops they felt very insecure in teaching mathematics. When interviewed at the Center following the workshops, they all felt that they could teach mathematics with confidence. But, what happens to the teachers who do not attend such workshops?" Edwards stated that many small colleges do not offer teaching methods courses specific to mathematics. He added that "too often, new teachers are simply given a textbook and no guidance on effective teaching methods."

Implications of the Results

In considering performance, Wilson noted that performance expectations must be based upon what is being taught in the schools: "If performance is satisfactory and the topic is being taught in the schools, that's a positive sign. If performance is lower on topics that are not emphasized, that's understandable. But there is cause for alarm in areas in which performance is not up to par, but which are areas that we are trying to emphasize in the curriculum."

In reviewing the NAEP results, the group addressed itself to the questions used to structure the knowledge and skill sections of the assessment:

Mathematical knowledge

1. *How well can students recall and recognize facts, definitions and symbols?*

Mathematical skill

1. *How well can students perform computations, including computations with whole numbers, integers, fractions, decimals, percents, ratios and proportions?*
2. *How well can students make measurements?*
3. *How well can students read graphs and tables?*
4. *How well can students perform geometric manipulations like constructions and visualizations?*
5. *How well can students perform algebraic manipulations?*
6. *How well can students estimate the*

answers to computations and measurements?

The panel was generally satisfied with performance on items assessing knowledge. Carpenter observed, "Almost all kids are learning basic facts," pointing out that results on basic facts for 13- and 17-year-olds (averaging over 90%) are approaching the upper limit (100%). Although 9-year-olds' performance was considerably lower on multiplication and division facts than on addition and subtraction facts, panel members felt it was generally consistent with the attention paid to multiplication and division at that age level. Lower performance on some terminology items — for example, those involving least common multiples and common factors for 13- and 17-year-olds and the name of a sphere for 9-year-olds — were not seen as cause for concern, since these do not receive a great deal of emphasis in the schools. While agreeing that a certain amount of basic vocabulary is essential, the group emphasized that there are limits to productivity in teaching vocabulary and felt that some effort might be better expended on other topics. Wilson summarized the group's feeling about knowledge achievement: "Knowledge levels reflect the emphasis placed on topics in the mathematics curriculum, and performance is generally very satisfactory."

In regard to computational skill, the panel felt that achievement on whole number computation items was adequate. Carpenter stated, "Performance with whole numbers is pretty good; computation with other types of numbers falls off."

Facility with decimal computation will probably be of increased importance in the future, especially as hand-held calculators and the metric system come into more widespread use. Recognizing that decimals are not presently in the curriculum for 9-year-olds, the panel felt that in consequence the low performance of that group is understandable. These results will, however, serve as a baseline measure for use in determining any future changes in achievement levels. For 9-year-olds, computations with money are frequently treated as whole number computations even though decimals are involved; thus, it is not surprising that this age group did better on money problems than on items showing decimals alone. For 13- and 17-year-olds, the panel did not feel that performance on decimal computation was satisfactory and noted it as an area of concern.

Panel members generally felt teenagers' achievement on computation with fractions was not adequate, but were concerned about the possibility of overemphasis on rote drill in teaching fractions. Carpenter stated, "Often children are taught fractional computations in rote drill and have little or no understanding of what fractions are. We need to promote understanding of fractions rather than rote, mechanical application of algorithms."

Skill in computing with percents was seen as a critical topic. Carpenter observed, "Percent exercises usually involve computations with decimals — on which performance was not high — so students may be starting with a handicap in dealing with them." Noting that many of the percent items were not difficult if one had an understanding of the concept of percent (for example, 30 is what percent of 60?), the group felt that more emphasis must be placed on understanding and working with it in a problem-solving context. The panel felt that teachers often teach students the algorithms for each of the "cases" associated with percent and that students memorize (and subsequently forget) the situations in which each formula is appropriate. Wilson said, "The primary emphasis in teaching percent in recent years, focusing on mechanical application of computational skills, has not led to adequate performance."

Performance in making measurements generally was regarded as satisfactory. Although some difficulties are seen in conversions between units, the general consensus was that the emphasis being placed upon it is probably sufficient. Emphasis on other aspects of measurement, particularly perimeter, area and volume, may not be sufficient. The current

attention to metrication appears to have produced adequate knowledge of the metric terminology included in the NAEP assessment.

The assessment provides evidence that students do not "think through" measurement problems. For example, although NAEP data show that at least 60% at each age level understood that the sides of a square are equal in length and about half of the 13-year-olds and 70% of the 17-year-olds calculated the area of a rectangle correctly when given the length and width, only 12% of the 13-year-olds and 44% of the 17-year-olds figured out how to find the area of a square when the dimension of only one side was given. Carpenter said, "Youngsters don't think about problems. They search for ways to apply the mechanical approaches that they have been taught, and if anything out of the ordinary appears in the problem, they are at a loss."

Geometry skills measured by the assessment are largely intuitive in nature, and the panel felt students' skills, on the whole, were adequate. The group also found skills at reading graphs and tables to be satisfactory, but were concerned that performance declined when interpretations or application of information read from the graph or table were required.

The level of algebra skills was viewed as somewhat disheartening by the panel, in light of the fact that about two-thirds of the 17-year-olds reported taking one year of Algebra I. Carpenter, noting that nearly all of the items included in the assessment would typically be part of a first-year algebra course, stated, "Items that require the use of algebra were generally answered correctly by 20—25% of the students; however, about two-thirds of them reported they had taken a year of Algebra I and 30% said they had taken one year of Algebra II. It seems that in this area, students in a number of instances are failing to meet course objectives."

The panel felt that results on many of the estimation items indicate a lack of understanding of the magnitude of numbers. They found it particularly striking that only 24% of the 13-year-olds and 37% of the 17-year-olds realized that 12/13 + 7/8 equals approximately 2. Dilworth stated that many students apparently do not understand the concept of fractions and mechanically tried to add the numbers in the limited time available, thus selecting unreasonable answers such as 19 and 21. Carl noted that "although students may have skill in using distinct algorithms, the depth of understanding of the concepts required to effectively apply skills in various situations appears to be missing." She also observed that, when asked to estimate, many students tried to calculate answers exactly and then round them off, thus defeating the purpose of estimation.

The panel agreed unanimously that estimation skills, although usually included in the curriculum, are often treated lightly, if at all, and that they should receive more emphasis than is presently the case. Dilworth stated that "estimation is a distinct cognitive operation of a different character than computational skills." Wilson agreed: "Estimation skills are highly important, but the emphasis on mechanical, algorithmic skills seen in much of the back to basics movement is antithetical to the processes required for success in estimating."

In examining results for population groups, panel members expressed some surprise that differences between various groups were similar for each of the four cognitive process levels assessed (knowledge, skill, understanding and application). The only exception to this pattern is for 17-year-old males and females. At this age, males did somewhat better than females on application items, while their performance was more similar on knowledge and skill items.

The interpretive panel acknowledged that many factors have an influence on discrepancies in group performance and felt that not enough information was available to single out particular ones. They did feel, however,

that the data present strong evidence that inequities in American education still remain.

How well are students doing in mathematical knowledge and skill? Results are mixed. While performance is generally satisfactory for knowledge and whole number computational skills, achievement on computing with decimals and percent, interpreting data read from graphs and tables, algebraic manipulation and estimating is below desired levels. Students do not appear to be very successful in thinking through mathematical tasks or in dealing with situations different from those they are accustomed to.

The consultants felt that narrowing the curriculum in response to the back to the basics movement has resulted in insufficient attention to higher-level processes, including problem solving. In emphasizing lower-level knowledge and skills, the present curriculum tends to lose sight of the fact that skills should not necessarily be ends in themselves, but should also be regarded as tools to be applied in problem-solving situations. In addition, ability with higher-level processes, such as understanding and "thinking through" problems, can often assist one in performing skill-level tasks.

The sequential nature of the mathematics curriculum may lead people to believe that students cannot go on to many topics and processes until they have completely mastered the preceding steps. This belief may result in a lack of student exposure to a broad range of concepts and processes. Carl noted that schools in Houston, Texas, have instituted a paced program; teachers and students move through the curriculum at a set speed. "Children can often perform better because they have had experiences in a number of areas. We can introduce division before children have mastered subtraction, because subtraction can be reinforced in division lessons."

Mathematics education may be influenced by recent increases in testing "basic" subjects. Noting that knowledge and computational skills are relatively easy to teach and to test, Edwards said, "There is a danger that increased concern over test scores may lead to concentration on training students to answer questions on tests, and other important concepts may not be explored."

In addition, some teachers may not be adequately prepared to teach understanding of concepts and may fall back on drill in lower-level cognitive processes. Carpenter stated, "People are teaching skills and expecting that facility with skills will automatically transfer to problem-solving ability. But the evidence shows that that's just not so."

Recommendations

In light of the data reported by NAEP, the panel felt that a number of recommendations are in order:

1. Performance on knowledge and whole number computation is satisfactory; more attention is not needed in these areas.

2. The remedy for areas in which performance is not satisfactory should *not* be to expand rote drill and mechanistic approaches to a larger number of topics. Teaching approaches should strive to promote understanding of concepts and to link skills to problem-solving activities.

3. Success on tests stressing low-level cognitive processes should not be the only criterion used to measure the effectiveness of mathematics programs. Too often the test then becomes the objective, and important areas that require more time and effort to teach fall by the wayside.

4. There is a need for more consistent and comprehensive teacher education in the mathematics area. More attention should be devoted to preparing teachers to teach concepts and prob-

lem solving so they will not tend to rely as heavily on drill and memorization work.

5. There is a need for further research into causes and potential remedies for the disparities in group performance that presently exist.

APPENDIX A

DEFINITIONS OF REPORTING GROUPS

National Assessment examines results for various groups of people within the national population. These groups are defined as follows.

Background Variables

Region

The country has been divided into four regions: Northeast, Southeast, Central and West. States included in each region are shown on the following map.

Sex

Results are reported for males and females.

Race

Results are presented for blacks, whites and Hispanos.

Level of Parental Education

Three categories of parental-education levels are defined by National Assessment, based on students' reports. These categories are: those whose parents did not graduate from high school, those who have at least one parent who graduated from high school and those who have at least one parent who has had some post-high-school education.

Type of Community

Communities in this category are defined by an occupational profile of the area served by a school as well as by the size of the community in which the school is located.

Advantaged-urban communities. Students in this group attend schools in or around cities having a population greater than 200,000 where a high proportion of the residents are in professional or managerial positions.

Disadvantaged-urban communities. Students in this group attend schools in or around cities having a population greater than 200,000 where a high proportion of the residents are on welfare or are not regularly employed.

Extreme-rural communities. Students in this group attend schools in areas having a population under 10,000 where most of the residents are farmers or farm workers.

Size of Community

Big cities. Students in this group attend schools within the city limits of cities having a 1970 census population over 200,000.

Fringes around big cities. Students in this group attend schools within metropolitan areas (1970 U.S. Bureau of the Census urbanized areas) served by cities having a population greater than 200,000, but outside the city limits.

Medium cities. Students in this group attend schools in cities having a population between 25,000 and 200,000, not classified in the fringes-around-big-cities category.

Smaller places. Students in this group attend schools in communities having a population less than 25,000, not classified in the fringes-around-big-cities category.

Grade Level

Results are categorized for 9-year-olds in the 3rd or 4th grade, 13-year-olds in the 7th or 8th grade, and 17-year-olds in the 10th, 11th or 12th grade.

Mathematics Courses Taken by 17-Year-Olds

Seventeen-year-olds were asked about their mathematics course-taking experiences. Five levels of mathematics-course experience were defined: (1) those who had not taken Algebra I or any more advanced mathematics courses; (2) those who had taken Algebra I but no more advanced courses; (3) those who had taken Geometry but not Algebra II or any more advanced courses; (4) those who had taken Algebra II but no more advanced courses (Trigonometry, Probability and Statistics, or Pre-calculus); (5) those who had taken Algebra II and at least one more advanced course — Trigonometry, Probability and Statistics, or Pre-calculus. A student was said to have taken a course only if he or she had taken it for one-half year or more.

APPENDIX B

GROUP DIFFERENCES FROM THE NATION FOR VARIOUS SETS OF MATHEMATICS KNOWLEDGE AND SKILLS ITEMS

TABLE B-1. Group Differences From the Nation for Various Sets of Mathematical Knowledge and Skills Items, Age 9

	All Knowledge Items	All Skills Items	Computation Items	Measurement Items	Read Graphs & Tables Items	Geometric Manipulation Items	Algebraic Manipulation Items
Number of items	161	137	48	30	16	15	19
National percentage	65.9	43.3	35.4	52.9	58.9	44.4	39.4
Region							
Northeast	3.7*	3.3*	3.9*	2.9*	2.9*	1.5	5.7*
Southeast	- 4.4*	- 4.3*	- 3.9*	- 4.6*	- 5.7*	- 3.9*	- 4.4*
Central	2.8*	2.8*	1.7*	3.9*	4.7*	2.4*	2.8*
West	- 2.7*	- 2.5*	- 2.0*	- 3.0*	- 3.0*	- 0.7	- 4.7*
Sex							
Male	- 0.7*	- 0.2	- 1.2*	1.0*	- 1.4*	0.9*	- 0.6
Female	0.7*	0.2	1.2*	- 1.0*	1.4*	- 0.9*	0.6
Race							
White	2.4*	2.3*	1.9*	2.5*	3.3*	2.3*	2.4*
Black	-11.0*	-10.8*	- 8.8*	-11.3*	-15.2*	-10.8*	-11.8*
Hispanic	- 9.2*	- 7.9*	- 6.1*	- 9.6*	-11.4*	- 7.3*	- 7.4*
Parental education							
Post high school	5.3*	5.0*	4.7*	4.7*	7.1*	4.1*	5.6*
Graduated high school	0.5	0.5	0.6	0.5	0.3	- 0.2	0.9
Not graduated high school	- 8.7*	- 7.7*	- 7.4*	- 5.8*	-13.3*	- 6.6*	- 8.3*
Type of community							
Advantaged urban	8.7*	8.9*	9.3*	8.6*	9.2*	6.9*	10.7*
Disadvantaged urban	-10.6*	- 8.9*	- 8.1*	- 7.5*	-14.2*	- 6.7*	-10.5*
Extreme rural	- 4.5*	- 3.3*	- 2.9	- 3.0	- 5.5*	- 2.4	- 4.0*
Size of community							
Fringes around big cities	4.3*	3.6*	3.7*	3.6*	4.2*	3.3*	3.8*
Big cities	- 4.3*	- 3.3*	- 2.8*	- 2.9*	- 5.4*	- 3.3*	- 2.7*
Medium cities	1.2	0.1	- 0.4	0.6	- 0.4	0.3	0.8
Smaller places	- 0.3	- 0.3	- 0.4	- 0.5	0.5	- 0.2	- 0.7
Grade							
3	-12.2*	-10.4*	-12.4*	- 8.7*	-12.3*	- 5.5*	-12.2*
4	4.3*	3.6*	4.2*	3.1*	4.2*	2.0*	4.2*

**Indicates mean percentages significantly different from the nation at the .05 level.*

TABLE B-2. Group Differences From the Nation for Various Sets of Mathematical Knowledge and Skills Items, Age 13

	All Knowledge Items	All Skills Items	Computation Items	Measurement Items	Read Graphs & Tables Items	Geometric Manipulation Items	Algebraic Manipulation Items	Estimation Items
Number of items	147	272	129	32	27	18	43	22
National percentage	66.9	51.9	51.7	54.8	68.8	45.8	52.2	32.8
Region								
Northeast	3.5*	5.0*	6.2*	3.9*	4.6*	1.8	4.8*	2.6*
Southeast	-5.0*	-6.5*	-6.9*	-7.9*	-5.7*	-4.2*	-7.1*	-4.4*
Central	2.6*	2.6*	2.0	4.1*	1.7	2.6*	3.5*	2.7*
West	-1.9*	-1.8	-2.0	-1.3	-1.2	-0.7	-2.3	-1.7
Sex								
Male	0.1	-0.6*	-1.7*	2.4*	-0.2	0.7*	-0.9*	1.4*
Female	-0.1	0.6*	1.8*	-2.4*	0.3	-0.7*	0.9*	-1.4*
Race								
White	2.9*	3.3*	3.3*	4.1*	3.3*	2.4*	3.8*	2.4*
Black	-14.0*	-16.8*	-16.4*	-21.2*	-16.6*	-12.4*	-18.6*	-12.9*
Hispanic	-10.9*	-12.0*	-12.5*	-13.2*	-11.4*	-6.4*	-14.0*	-8.2*
Parental education								
Post high school	5.9*	6.9*	7.3*	7.4*	6.3*	4.8*	7.0*	6.1*
Graduated high school	-0.6	-0.9*	-1.0	-0.8	-0.7	-0.8	-0.5	-1.6*
Not graduated high school	-7.5*	-8.9*	-9.3*	-10.0*	-7.9*	-6.1*	-9.5*	-7.8*
Type of community								
Advantaged urban	7.3*	9.0*	9.6*	10.4*	6.0*	6.6*	9.7*	7.5*
Disadvantaged urban	-10.3*	-12.8*	-12.8*	-13.6*	-14.8*	-8.1*	-15.5*	-8.2*
Extreme rural	-4.4*	-4.7*	-4.9	-3.8	-5.0*	-5.5*	-4.5	-4.2*
Size of community								
Fringes around big cities	3.6*	4.8*	5.5*	4.7*	2.8*	4.0*	4.9*	3.5*
Big cities	-2.5*	-3.1*	-2.8*	-4.2*	-3.2*	-2.2	-4.0*	-1.8
Medium cities	3.9*	5.2*	5.3	4.4	6.0*	4.0	6.1*	4.0*
Smaller places	-1.2	-1.8*	-2.4*	-1.0	-0.9	-1.6*	-1.6	-1.6*
Grade								
7	-8.8*	-11.6*	-14.6*	-9.0*	-9.1*	-6.2*	-11.2*	-7.2*
8	3.9*	5.0*	6.1*	4.1*	4.1*	2.8*	5.0*	3.2*

**Indicates mean percentages significantly different from the nation at the .05 level.*

TABLE B-3. Group Differences From the Nation for Various Sets of Mathematical Knowledge and Skills Items, Age 17

	All Knowledge Items	All Skills Items	Computation Items	Measurement Items	Read Graphs & Tables Items	Geometric Manipulation Items	Algebraic Manipulation Items	Estimation Items
Number of items	140	273	127	21	29	15	59	22
National percentage	71.7	59.0	57.2	56.7	72.6	54.8	40.0	49.6
Region								
Northeast	2.5*	3.3*	3.9*	2.1	0.8	0.5	4.8*	2.8*
Southeast	- 3.5*	- 4.6*	- 4.3*	- 6.2*	- 3.8*	- 6.9*	- 4.6*	- 4.5*
Central	2.5*	2.6*	2.2*	4.4*	3.5*	5.0*	1.6	3.3*
West	- 2.6*	- 2.7*	- 2.9*	- 2.7*	- 2.2*	- 1.2	- 2.8*	- 3.3*
Sex								
Male	1.1*	1.3*	0.5*	4.9*	0.6*	3.1*	0.9*	3.3*
Female	- 1.1*	- 1.2*	- 0.4*	- 4.7*	- 0.6	- 3.0*	- 0.9*	- 3.2*
Race								
White	2.6*	2.9*	2.9*	3.7*	2.9*	3.1*	2.5*	3.3*
Black	-15.5*	-17.6*	-17.6*	-23.5*	-16.6*	-18.9*	-14.9*	-19.5*
Hispanic	-11.8*	-12.0*	-12.3*	-13.2*	-10.8*	-13.0*	-11.5*	-11.6*
Parental education								
Post high school	5.5*	6.3*	6.2*	6.3*	5.1*	5.7*	7.0*	6.7*
Graduated high school	- 2.5*	- 3.0*	- 2.8*	- 2.4*	- 2.4*	- 2.2*	- 3.8*	- 3.4*
Not graduated high school	- 9.4*	-10.8*	-10.9*	-11.8*	- 8.8*	-10.8*	-11.2*	-11.6*
Type of community								
Advantaged urban	7.6*	9.8*	9.7*	8.3*	6.8*	9.8*	11.5*	10.5*
Disadvantaged urban	-12.5*	-13.4*	-13.3*	-17.7*	-13.1*	-13.6*	-11.6*	-14.4*
Extreme rural	- 1.1	- 2.5*	- 2.9*	0.2	0.3	- 1.2	- 4.8*	- 1.7
Size of community								
Fringes around big cities	2.7*	3.1*	3.0*	1.9	1.9*	3.2*	4.4*	3.2*
Big cities	- 2.7*	- 2.1	- 2.2	- 3.4*	- 2.6*	- 2.4	- 0.8	- 2.9*
Medium cities	3.2	5.5*	6.4*	3.3	3.5*	3.9	5.6*	5.4*
Smaller places	- 0.7	- 1.6*	- 1.6*	0.2	- 0.3	- 1.2	- 2.9*	- 1.3
Grade								
10	-11.5*	-13.3*	-13.9*	-12.1*	-13.4*	-11.5*	-12.9*	-13.5*
11	2.2*	2.3*	2.4*	2.4*	2.4*	2.1*	1.9*	2.4*
12	3.0*	4.4*	4.0*	3.1*	4.4*	3.1*	6.2*	4.3*
Courses taken								
<Algebra I	-13.9*	-17.2*	-19.5*	-14.1*	-10.8*	-14.0*	-17.6*	-16.5*
Algebra I	- 5.5*	- 6.2*	- 5.3*	- 6.3*	- 2.0*	- 8.1*	- 9.2*	- 7.4*
Geometry	4.3*	1.8*	2.7*	2.2*	3.3*	5.7*	- 2.3*	1.8*
Algebra II	8.2*	10.8*	12.0*	8.8*	6.2*	7.5*	12.5*	10.0*
>Algebra II	15.9*	20.7*	20.2*	19.7*	11.1*	16.8*	27.4*	21.3*

**Indicates mean percentages significantly different from the nation at the .05 level.*

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